1.2 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. An ordered pair (a, b) is a ________ of an equation in x and y if the equation is true when a is substituted for x and b is substituted for y.
2. The set of all solution points of an equation is the ________ of the equation.
3. The points at which a graph intersects or touches an axis are called the ________ of the graph.
4. A graph is symmetric with respect to the ________ if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
5. The equation $(x - h)^2 + (y - k)^2 = r^2$ is the standard form of the equation of a ________ with center ________ and radius ________.
6. When you construct and use a table to solve a problem, you are using a ________ approach.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, determine whether each point lies on the graph of the equation.

Equation	*Points*	
1. $y = \sqrt{x+4}$	(a) $(0, 2)$	(b) $(5, 3)$
2. $y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
3. $y = 4 - \lvert x - 2 \rvert$	(a) $(1, 5)$	(b) $(6, 0)$
4. $y = \frac{1}{3}x^3 - 2x^2$	(a) $\left(2, -\frac{16}{3}\right)$	(b) $(-3, 9)$

In Exercises 5–8, complete the table. Use the resulting solution points to sketch the graph of the equation.

5. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y					
(x, y)					

6. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y					
(x, y)					

7. $y = x^2 - 3x$

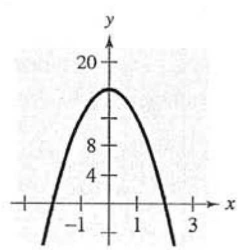
x	-1	0	1	2	3
y					
(x, y)					

8. $y = 5 - x^2$

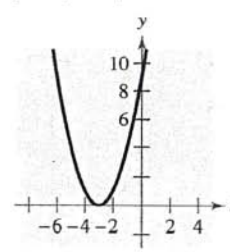
x	-2	-1	0	1	2
y					
(x, y)					

In Exercises 9–20, find the *x*- and *y*-intercepts of the graph of the equation.

9. $y = 16 - 4x^2$



10. $y = (x + 3)^2$



11. $y = 5x - 6$

12. $y = 8 - 3x$

13. $y = \sqrt{x + 4}$

14. $y = \sqrt{2x - 1}$

15. $y = \lvert 3x - 7 \rvert$

16. $y = -\lvert x + 10 \rvert$

17. $y = 2x^3 - 4x^2$

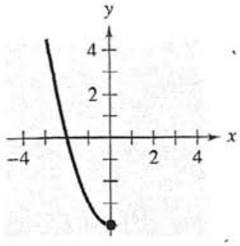
18. $y = x^4 - 25$

19. $y^2 = 6 - x$

20. $y^2 = x + 1$

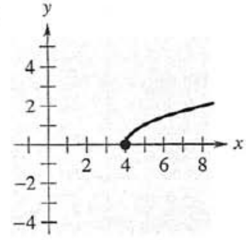
In Exercises 21–24, assume that the graph has the indicated type of symmetry. Sketch the complete graph of the equation. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

21.



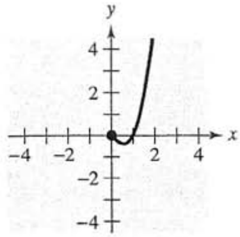
y-axis symmetry

22.



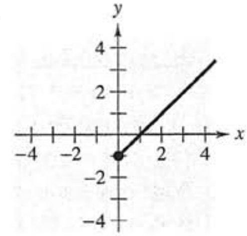
x-axis symmetry

23.



Origin symmetry

24.



y-axis symmetry

In Exercises 25–32, use the algebraic tests to check for symmetry with respect to both axes and the origin.

25. $x^2 - y = 0$
26. $x - y^2 = 0$
27. $y = x^3$
28. $y = x^4 - x^2 + 3$
29. $y = \dfrac{x}{x^2 + 1}$
30. $y = \dfrac{1}{x^2 + 1}$
31. $xy^2 + 10 = 0$
32. $xy = 4$

In Exercises 33–44, use symmetry to sketch the graph of the equation.

33. $y = -3x + 1$
34. $y = 2x - 3$
35. $y = x^2 - 2x$
36. $y = -x^2 - 2x$
37. $y = x^3 + 3$
38. $y = x^3 - 1$
39. $y = \sqrt{x - 3}$
40. $y = \sqrt{1 - x}$
41. $y = |x - 6|$
42. $y = 1 - |x|$
43. $x = y^2 - 1$
44. $x = y^2 - 5$

In Exercises 45–56, use a graphing utility to graph the equation. Use a standard setting. Approximate any intercepts.

45. $y = 3 - \frac{1}{2}x$
46. $y = \frac{2}{3}x - 1$
47. $y = x^2 - 4x + 3$
48. $y = x^2 + x - 2$
49. $y = \dfrac{2x}{x - 1}$
50. $y = \dfrac{4}{x^2 + 1}$
51. $y = \sqrt[3]{x}$
52. $y = \sqrt[3]{x + 1}$
53. $y = x\sqrt{x + 6}$
54. $y = (6 - x)\sqrt{x}$
55. $y = |x + 3|$
56. $y = 2 - |x|$

In Exercises 57–64, write the standard form of the equation of the circle with the given characteristics.

57. Center: $(0, 0)$; radius: 4
58. Center: $(0, 0)$; radius: 5
59. Center: $(2, -1)$; radius: 4
60. Center: $(-7, -4)$; radius: 7
61. Center: $(-1, 2)$; solution point: $(0, 0)$
62. Center: $(3, -2)$; solution point: $(-1, 1)$
63. Endpoints of a diameter: $(0, 0)$, $(6, 8)$
64. Endpoints of a diameter: $(-4, -1)$, $(4, 1)$

In Exercises 65–70, find the center and radius of the circle, and sketch its graph.

65. $x^2 + y^2 = 25$
66. $x^2 + y^2 = 16$
67. $(x - 1)^2 + (y + 3)^2 = 9$
68. $x^2 + (y - 1)^2 = 1$
69. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$
70. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

71. ***Depreciation*** A manufacturing plant purchases a new molding machine for \$225,000. The depreciated value y (reduced value) after t years is given by $y = 225{,}000 - 20{,}000t, 0 \le t \le 8$. Sketch the graph of the equation.

72. ***Consumerism*** You purchase a jet ski for \$8100. The depreciated value y after t years is given by $y = 8100 - 929t,\ 0 \le t \le 6$. Sketch the graph of the equation.

73. ***Geometry*** A regulation NFL playing field (including the end zones) of length x and width y has a perimeter of $346\frac{2}{3}$ or $\frac{1040}{3}$ yards.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $y = \dfrac{520}{3} - x$ and its area is $A = x\left(\dfrac{520}{3} - x\right)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation NFL playing field and compare your findings with the results of part (d).

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

74. ***Geometry*** A soccer playing field of length x and width y has a perimeter of 360 meters.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $w = 180 - x$ and its area is $A = x(180 - x)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation Major League Soccer field and compare your findings with the results of part(d).

Model It

75. ***Population Statistics*** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1920 to 2000. (Source: U.S. National Center for Health Statistics)

Year	Life expectancy, y
1920	54.1
1930	59.7
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4
2000	77.0

A model for the life expectancy during this period is

$$y = -0.0025t^2 + 0.574t + 44.25, \quad 20 \le t \le 100$$

where y represents the life expectancy and t is the time in years, with $t = 20$ corresponding to 1920.

(a) Sketch a scatter plot of the data.

(b) Graph the model for the data and compare the scatter plot and the graph.

(c) Determine the life expectancy in 1948 both graphically and algebraically.

(d) Use the graph of the model to estimate the life expectancies of a child for the years 2005 and 2010.

(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

76. ***Electronics*** The resistance y (in ohms) of 1000 feet of solid copper wire at 68 degrees Fahrenheit can be approximated by the model $y = \dfrac{10,770}{x^2} - 0.37,\ 5 \le x \le 100$ where x is the diameter of the wire in mils (0.001 inch). (Source: American Wire Gage)

(a) Complete the table.

x	5	10	20	30	40	50
y						

x	60	70	80	90	100
y					

(b) Use the table of values in part (a) to sketch a graph of the model. Then use your graph to estimate the resistance when $x = 85.5$.

(c) Use the model to confirm algebraically the estimate you found in part (b).

(d) What can you conclude in general about the relationship between the diameter of the copper wire and the resistance?

Synthesis

True or False? **In Exercises 77 and 78, determine whether the statement is true or false. Justify your answer.**

77. A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.

78. A graph of an equation can have more than one y-intercept.

79. ***Think About It*** Suppose you correctly enter an expression for the variable y on a graphing utility. However, no graph appears on the display when you graph the equation. Give a possible explanation and the steps you could take to remedy the problem. Illustrate your explanation with an example.

80. ***Think About It*** Find a and b if the graph of $y = ax^2 + bx^3$ is symmetric with respect to (a) the y-axis and (b) the origin. (There are many correct answers.)

Skills Review

81. Identify the terms: $9x^5 + 4x^3 - 7$.

82. Rewrite the expression using exponential notation.

$-(7 \times 7 \times 7 \times 7)$

In Exercises 83–88, simplify the expression.

83. $\sqrt{18x} - \sqrt{2x}$

84. $\sqrt[4]{x^5}$

85. $\dfrac{70}{\sqrt{7x}}$

86. $\dfrac{55}{\sqrt{20} - 3}$

87. $\sqrt[6]{t^2}$

88. $\sqrt[3]{\sqrt{y}}$

1.3 Exercises

VOCABULARY CHECK:

In Exercises 1–6, fill in the blanks.

1. The simplest mathematical model for relating two variables is the ________ equation in two variables $y = mx + b$.
2. For a line, the ratio of the change in y to the change in x is called the ________ of the line.
3. Two lines are ________ if and only if their slopes are equal.
4. Two lines are ________ if and only if their slopes are negative reciprocals of each other.
5. When the x-axis and y-axis have different units of measure, the slope can be interpreted as a ________.
6. The prediction method ________ ________ is the method used to estimate a point on a line that does not lie between the given points.
7. Match each equation of a line with its form.

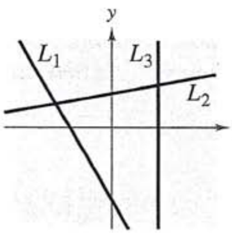
(a) $Ax + By + C = 0$	(i) Vertical line
(b) $x = a$	(ii) Slope-intercept form
(c) $y = b$	(iii) General form
(d) $y = mx + b$	(iv) Point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) Horizontal line

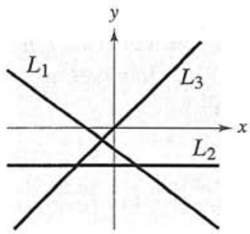
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1 and 2, identify the line that has each slope.

1. (a) $m = \frac{2}{3}$
   (b) m is undefined.
   (c) $m = -2$

2. (a) $m = 0$
   (b) $m = -\frac{3}{4}$
   (c) $m = 1$

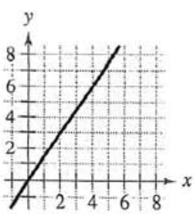




In Exercises 3 and 4, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.

Point	*Slopes*			
3. $(2, 3)$	(a) 0	(b) 1	(c) 2	(d) -3
4. $(-4, 1)$	(a) 3	(b) -3	(c) $\frac{1}{2}$	(d) Undefined

In Exercises 5–8, estimate the slope of the line.

5.


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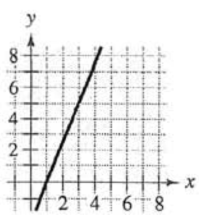

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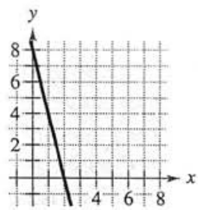

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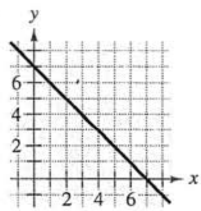


In Exercises 9–20, find the slope and y-intercept (if possible) of the equation of the line. Sketch the line.

9. $y = 5x + 3$
10. $y = x - 10$
11. $y = -\frac{1}{2}x + 4$
12. $y = -\frac{3}{2}x + 6$
13. $5x - 2 = 0$
14. $3y + 5 = 0$
15. $7x + 6y = 30$
16. $2x + 3y = 9$
17. $y - 3 = 0$
18. $y + 4 = 0$
19. $x + 5 = 0$
20. $x - 2 = 0$

In Exercises 21–28, plot the points and find the slope of the line passing through the pair of points.

21. $(-3, -2), (1, 6)$
22. $(2, 4), (4, -4)$
23. $(-6, -1), (-6, 4)$
24. $(0, -10), (-4, 0)$
25. $\left(\frac{11}{2}, -\frac{4}{3}\right), \left(-\frac{3}{2}, -\frac{1}{3}\right)$
26. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$
27. $(4.8, 3.1), (-5.2, 1.6)$
28. $(-1.75, -8.3), (2.25, -2.6)$

In Exercises 29–38, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

Point	*Slope*
29. $(2, 1)$	$m = 0$
30. $(-4, 1)$	m is undefined.
31. $(5, -6)$	$m = 1$
32. $(10, -6)$	$m = -1$
33. $(-8, 1)$	m is undefined.
34. $(-3, -1)$	$m = 0$
35. $(-5, 4)$	$m = 2$
36. $(0, -9)$	$m = -2$
37. $(7, -2)$	$m = \frac{1}{2}$
38. $(-1, -6)$	$m = -\frac{1}{2}$

In Exercises 39–50, find the slope-intercept form of the equation of the line that passes through the given point and has the indicated slope. Sketch the line.

Point	*Slope*
39. $(0, -2)$	$m = 3$
40. $(0, 10)$	$m = -1$
41. $(-3, 6)$	$m = -2$
42. $(0, 0)$	$m = 4$
43. $(4, 0)$	$m = -\frac{1}{3}$
44. $(-2, -5)$	$m = \frac{3}{4}$
45. $(6, -1)$	m is undefined.
46. $(-10, 4)$	m is undefined.
47. $\left(4, \frac{5}{2}\right)$	$m = 0$
48. $\left(-\frac{1}{2}, \frac{3}{2}\right)$	$m = 0$
49. $(-5.1, 1.8)$	$m = 5$
50. $(2.3, -8.5)$	$m = -\frac{5}{2}$

In Exercises 51–64, find the slope-intercept form of the equation of the line passing through the points. Sketch the line.

51. $(5, -1), (-5, 5)$ **52.** $(4, 3), (-4, -4)$

53. $(-8, 1), (-8, 7)$ **54.** $(-1, 4), (6, 4)$

55. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$ **56.** $(1, 1), \left(6, -\frac{2}{3}\right)$

57. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$ **58.** $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

59. $(1, 0.6), (-2, -0.6)$

60. $(-8, 0.6), (2, -2.4)$

61. $(2, -1), \left(\frac{1}{3}, -1\right)$

62. $\left(\frac{1}{5}, -2\right), (-6, -2)$

63. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

64. $(1.5, -2), (1.5, 0.2)$

In Exercises 65–68, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.

65. L_1: $(0, -1), (5, 9)$
L_2: $(0, 3), (4, 1)$

66. L_1: $(-2, -1), (1, 5)$
L_2: $(1, 3), (5, -5)$

67. L_1: $(3, 6), (-6, 0)$
L_2: $(0, -1), \left(5, \frac{7}{3}\right)$

68. L_1: $(4, 8), (-4, 2)$
L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

In Exercises 69–78, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.

Point	*Line*
69. $(2, 1)$	$4x - 2y = 3$
70. $(-3, 2)$	$x + y = 7$
71. $\left(-\frac{2}{3}, \frac{7}{8}\right)$	$3x + 4y = 7$
72. $\left(\frac{7}{8}, \frac{3}{4}\right)$	$5x + 3y = 0$
73. $(-1, 0)$	$y = -3$
74. $(4, -2)$	$y = 1$
75. $(2, 5)$	$x = 4$
76. $(-5, 1)$	$x = -2$
77. $(2.5, 6.8)$	$x - y = 4$
78. $(-3.9, -1.4)$	$6x + 2y = 9$

In Exercises 79–84, use the *intercept form* to find the equation of the line with the given intercepts. The intercept form of the equation of a line with intercepts $(a, 0)$ and $(0, b)$ is

$$\frac{x}{a} + \frac{y}{b} = 1, \ a \neq 0, \ b \neq 0.$$

79. x-intercept: $(2, 0)$
y-intercept: $(0, 3)$

80. x-intercept: $(-3, 0)$
y-intercept: $(0, 4)$

81. x-intercept: $\left(-\frac{1}{6}, 0\right)$
y-intercept: $\left(0, -\frac{2}{3}\right)$

82. x-intercept: $\left(\frac{2}{3}, 0\right)$
y-intercept: $(0, -2)$

83. Point on line: $(1, 2)$
x-intercept: $(c, 0)$
y-intercept: $(0, c)$, $c \neq 0$

84. Point on line: $(-3, 4)$
x-intercept: $(d, 0)$
y-intercept: $(0, d)$, $d \neq 0$

Graphical Interpretation **In Exercises 85–88, identify any relationships that exist among the lines, and then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that the slope appears visually correct—that is, so that parallel lines appear parallel and perpendicular lines appear to intersect at right angles.**

85. (a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$

86. (a) $y = \frac{2}{3}x$ (b) $y = -\frac{3}{2}x$ (c) $y = \frac{2}{3}x + 2$

87. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = 2x - 4$

88. (a) $y = x - 8$ (b) $y = x + 1$ (c) $y = -x + 3$

In Exercises 89–92, find a relationship between x and y such that (x, y) is equidistant (the same distance) from the two points.

89. $(4, -1), (-2, 3)$

90. $(6, 5), (1, -8)$

91. $\left(3, \frac{5}{2}\right), (-7, 1)$

92. $\left(-\frac{1}{2}, -4\right), \left(\frac{7}{2}, \frac{5}{4}\right)$

93. ***Sales*** The following are the slopes of lines representing annual sales y in terms of time x in years. Use the slopes to interpret any change in annual sales for a one-year increase in time.

(a) The line has a slope of $m = 135$.

(b) The line has a slope of $m = 0$.

(c) The line has a slope of $m = -40$.

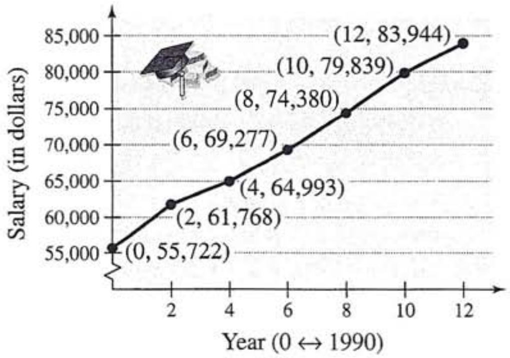
94. ***Revenue*** The following are the slopes of lines representing daily revenues y in terms of time x in days. Use the slopes to interpret any change in daily revenues for a one-day increase in time.

(a) The line has a slope of $m = 400$.

(b) The line has a slope of $m = 100$.

(c) The line has a slope of $m = 0$.

95. ***Average Salary*** The graph shows the average salaries for senior high school principals from 1990 through 2002. (Source: Educational Research Service)

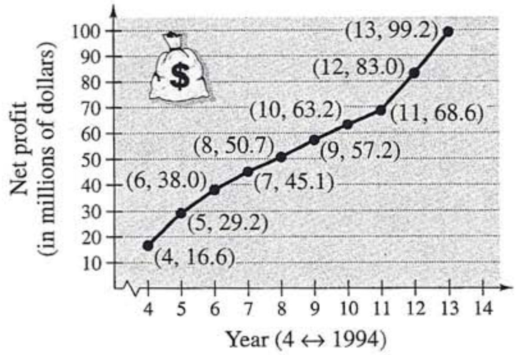


(a) Use the slopes to determine the time periods in which the average salary increased the greatest and the least.

(b) Find the slope of the line segment connecting the years 1990 and 2002.

(c) Interpret the meaning of the slope in part (b) in the context of the problem.

96. ***Net Profit*** The graph shows the net profits (in millions) for Applebee's International, Inc. for the years 1994 through 2003. (Source: Applebee's International, Inc.)



(a) Use the slopes to determine the years in which the net profit showed the greatest increase and the least increase.

(b) Find the slope of the line segment connecting the years 1994 and 2003.

(c) Interpret the meaning of the slope in part (b) in the context of the problem.

97. ***Road Grade*** You are driving on a road that has a 6% uphill grade (see figure). This means that the slope of the road is $\frac{6}{100}$. Approximate the amount of vertical change in your position if you drive 200 feet.

98. ***Road Grade*** From the top of a mountain road, a surveyor takes several horizontal measurements x and several vertical measurements y, as shown in the table (x and y are measured in feet).

x	300	600	900	1200	1500	1800	2100
y	-25	-50	-75	-100	-125	-150	-175

(a) Sketch a scatter plot of the data.

(b) Use a straightedge to sketch the line that you think best fits the data.

(c) Find an equation for the line you sketched in part (b).

(d) Interpret the meaning of the slope of the line in part (c) in the context of the problem.

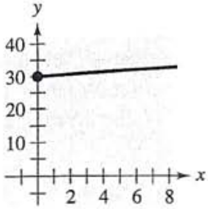
(e) The surveyor needs to put up a road sign that indicates the steepness of the road. For instance, a surveyor would put up a sign that states "8% grade" on a road with a downhill grade that has a slope of $-\frac{8}{100}$. What should the sign state for the road in this problem?

Rate of Change **In Exercises 99 and 100, you are given the dollar value of a product in 2005 and the rate at which the value of the product is expected to change during the next 5 years. Use this information to write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 5$ represent 2005.)**

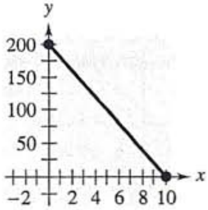
	2005 Value	*Rate*
99.	\$2540	\$125 decrease per year
100.	\$156	\$4.50 increase per year

Graphical Interpretation **In Exercises 101–104, match the description of the situation with its graph. Also determine the slope and y-intercept of each graph and interpret the slope and y-intercept in the context of the situation. [The graphs are labeled (a), (b), (c), and (d).]**

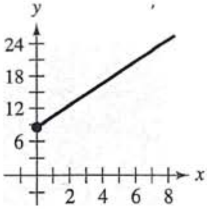
(a)



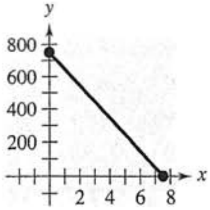
(b)



(c)



(d)



101. A person is paying \$20 per week to a friend to repay a \$200 loan.

102. An employee is paid \$8.50 per hour plus \$2 for each unit produced per hour.

103. A sales representative receives \$30 per day for food plus \$0.32 for each mile traveled.

104. A computer that was purchased for \$750 depreciates \$100 per year.

105. ***Cash Flow per Share*** The cash flow per share for the Timberland Co. was \$0.18 in 1995 and \$4.04 in 2003. Write a linear equation that gives the cash flow per share in terms of the year. Let $t = 5$ represent 1995. Then predict the cash flows for the years 2008 and 2010. (Source: The Timberland Co.)

106. ***Number of Stores*** In 1999 there were 4076 J.C. Penney stores and in 2003 there were 1078 stores. Write a linear equation that gives the number of stores in terms of the year. Let $t = 9$ represent 1999. Then predict the numbers of stores for the years 2008 and 2010. Are your answers reasonable? Explain. (Source: J.C. Penney Co.)

107. ***Depreciation*** A sub shop purchases a used pizza oven for \$875. After 5 years, the oven will have to be replaced. Write a linear equation giving the value V of the equipment during the 5 years it will be in use.

108. ***Depreciation*** A school district purchases a high-volume printer, copier, and scanner for \$25,000. After 10 years, the equipment will have to be replaced. Its value at that time is expected to be \$2000. Write a linear equation giving the value V of the equipment during the 10 years it will be in use.

109. ***College Enrollment*** The Pennsylvania State University had enrollments of 40,571 students in 2000 and 41,289 students in 2004 at its main campus in University Park, Pennsylvania. (Source: *Penn State Fact Book*)

(a) Assuming the enrollment growth is linear, find a linear model that gives the enrollment in terms of the year t, where $t = 0$ corresponds to 2000.

(b) Use your model from part (a) to predict the enrollments in 2008 and 2010.

(c) What is the slope of your model? Explain its meaning in the context of the situation.

110. ***College Enrollment*** The University of Florida had enrollments of 36,531 students in 1990 and 48,673 students in 2003. (Source: University of Florida)

(a) What was the average annual change in enrollment from 1990 to 2003?

(b) Use the average annual change in enrollment to estimate the enrollments in 1994, 1998, and 2002.

(c) Write the equation of a line that represents the given data. What is its slope? Interpret the slope in the context of the problem.

(d) Using the results of parts (a)–(c), write a short paragraph discussing the concepts of *slope* and *average rate of change*.

111. ***Sales*** A discount outlet is offering a 15% discount on all items. Write a linear equation giving the sale price S for an item with a list price L.

112. ***Hourly Wage*** A microchip manufacturer pays its assembly line workers \$11.50 per hour. In addition, workers receive a piecework rate of \$0.75 per unit produced. Write a linear equation for the hourly wage W in terms of the number of units x produced per hour.

113. ***Cost, Revenue, and Profit*** A roofing contractor purchases a shingle delivery truck with a shingle elevator for \$36,500. The vehicle requires an average expenditure of \$5.25 per hour for fuel and maintenance, and the operator is paid \$11.50 per hour.

(a) Write a linear equation giving the total cost C of operating this equipment for t hours. (Include the purchase cost of the equipment.)

(b) Assuming that customers are charged \$27 per hour of machine use, write an equation for the revenue R derived from t hours of use.

(c) Use the formula for profit ($P = R - C$) to write an equation for the profit derived from t hours of use.

(d) Use the result of part (c) to find the break-even point—that is, the number of hours this equipment must be used to yield a profit of 0 dollars.

114. ***Rental Demand*** A real estate office handles an apartment complex with 50 units. When the rent per unit is \$580 per month, all 50 units are occupied. However, when the rent is \$625 per month, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear.

(a) Write the equation of the line giving the demand x in terms of the rent p.

(b) Use this equation to predict the number of units occupied when the rent is \$655.

(c) Predict the number of units occupied when the rent is \$595.

115. ***Geometry*** The length and width of a rectangular garden are 15 meters and 10 meters, respectively. A walkway of width x surrounds the garden.

(a) Draw a diagram that gives a visual representation of the problem.

(b) Write the equation for the perimeter y of the walkway in terms of x.

(c) Use a graphing utility to graph the equation for the perimeter.

(d) Determine the slope of the graph in part (c). For each additional one-meter increase in the width of the walkway, determine the increase in its perimeter.

116. ***Monthly Salary*** A pharmaceutical salesperson receives a monthly salary of \$2500 plus a commission of 7% of sales. Write a linear equation for the salesperson's monthly wage W in terms of monthly sales S.

117. ***Business Costs*** A sales representative of a company using a personal car receives \$120 per day for lodging and meals plus \$0.38 per mile driven. Write a linear equation giving the daily cost C to the company in terms of x, the number of miles driven.

118. ***Sports*** The median salaries (in thousands of dollars) for players on the Los Angeles Dodgers from 1996 to 2003 are shown in the scatter plot. Find the equation of the line that you think best fits these data. (Let y represent the median salary and let t represent the year, with $t = 6$ corresponding to 1996.) (Source: *USA TODAY*)

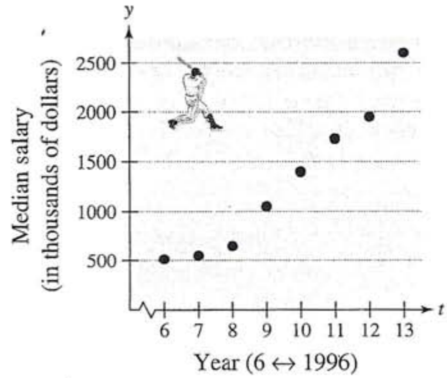


FIGURE FOR 118

Model It

119. ***Data Analysis: Cell Phone Suscribers*** The numbers of cellular phone suscribers y (in millions) in the United States from 1990 through 2002, where x is the year, are shown as data points (x, y). (Source: Cellular Telecommunications & Internet Association)

(1990, 5.3)
(1991, 7.6)
(1992, 11.0)
(1993, 16.0)
(1994, 24.1)
(1995, 33.8)
(1996, 44.0)
(1997, 55.3)
(1998, 69.2)
(1999, 86.0)
(2000, 109.5)
(2001, 128.4)
(2002, 140.8)

(a) Sketch a scatter plot of the data. Let $x = 0$ correspond to 1990.

(b) Use a straightedge to sketch the line that you think best fits the data.

(c) Find the equation of the line from part (b). Explain the procedure you used.

(d) Write a short paragraph explaining the meanings of the slope and y-intercept of the line in terms of the data.

(e) Compare the values obtained using your model with the actual values.

(f) Use your model to estimate the number of cellular phone suscribers in 2008.

120. ***Data Analysis: Average Scores*** An instructor gives regular 20-point quizzes and 100-point exams in an algebra course. Average scores for six students, given as data points (x, y) where x is the average quiz score and y is the average test score, are $(18, 87)$, $(10, 55)$, $(19, 96)$, $(16, 79)$, $(13, 76)$, and $(15, 82)$. [*Note:* There are many correct answers for parts (b)–(d).]

(a) Sketch a scatter plot of the data.

(b) Use a straightedge to sketch the line that you think best fits the data.

(c) Find an equation for the line you sketched in part (b).

(d) Use the equation in part (c) to estimate the average test score for a person with an average quiz score of 17.

(e) The instructor adds 4 points to the average test score of each student in the class. Describe the changes in the positions of the plotted points and the change in the equation of the line.

Synthesis

True or False? **In Exercises 121 and 122, determine whether the statement is true or false. Justify your answer.**

121. A line with a slope of $-\frac{5}{7}$ is steeper than a line with a slope of $-\frac{6}{7}$.

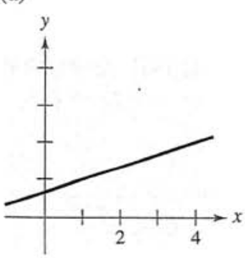
122. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

123. Explain how you could show that the points $A\,(2, 3)$, $B\,(2, 9)$, and $C\,(4, 3)$ are the vertices of a right triangle.

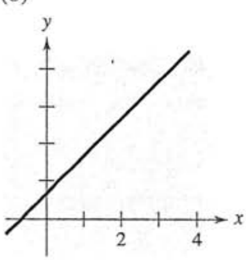
124. Explain why the slope of a vertical line is said to be undefined.

125. With the information shown in the graphs, is it possible to determine the slope of each line? Is it possible that the lines could have the same slope? Explain.

(a)



(b)



126. The slopes of two lines are -4 and $\frac{5}{2}$. Which is steeper? Explain.

127. The value V of a molding machine t years after it is purchased is

$$V = -4000t + 58{,}500, \quad 0 \le t \le 5.$$

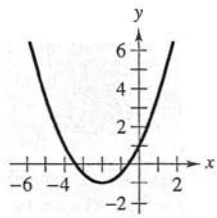
Explain what the V-intercept and slope measure.

128. ***Think About It*** Is it possible for two lines with positive slopes to be perpendicular? Explain.

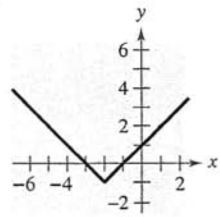
Skills Review

In Exercises 129–132, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

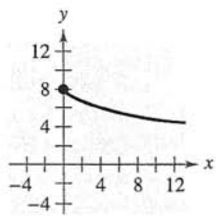
(a)



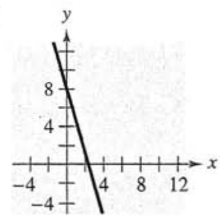
(b)



(c)



(d)



129. $y = 8 - 3x$

130. $y = 8 - \sqrt{x}$

131. $y = \frac{1}{2}x^2 + 2x + 1$

132. $y = |x + 2| - 1$

In Exercises 133–138, find all the solutions of the equation. Check your solution(s) in the original equation.

133. $-7(3 - x) = 14(x - 1)$

134. $\dfrac{8}{2x - 7} = \dfrac{4}{9 - 4x}$

135. $2x^2 - 21x + 49 = 0$

136. $x^2 - 8x + 3 = 0$

137. $\sqrt{x - 9} + 15 = 0$

138. $3x - 16\sqrt{x} + 5 = 0$

139. **Make a Decision** To work an extended application analyzing the numbers of bachelor's degrees earned by women in the United States from 1985 to 2002, visit this text's website at *college.hmco.com.* *(Data Source: U.S. Census Bureau)*

1.4 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. A relation that assigns to each element x from a set of inputs, or ________, exactly one element y in a set of outputs, or ________, is called a ________.
2. Functions are commonly represented in four different ways, ________, ________, ________, and ________.
3. For an equation that represents y as a function of x, the set of all values taken on by the ________ variable x is the domain, and the set of all values taken on by the ________ variable y is the range.
4. The function given by

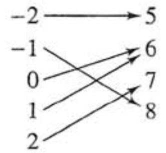
$$f(x) = \begin{cases} 2x - 1, & x < 0 \\ x^2 + 4, & x \geq 0 \end{cases}$$

is an example of a ________ function.
5. If the domain of the function f is not given, then the set of values of the independent variable for which the expression is defined is called the ________ ________.
6. In calculus, one of the basic definitions is that of a ________ ________, given by $\dfrac{f(x + h) - f(x)}{h}$, $h \neq 0$.

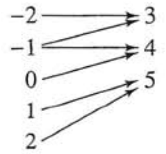
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, is the relationship a function?

1. *Domain* *Range*

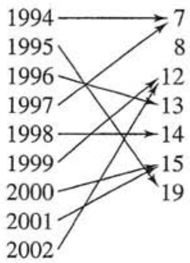


2. *Domain* *Range*



3. *Domain* *Range*

4. *Domain* (Year) *Range* (Number of North Atlantic tropical storms and hurricanes)



In Exercises 5–8, does the table describe a function? Explain your reasoning.

5.

Input value	−2	−1	0	1	2
Output value	−8	−1	0	1	8

6.

Input value	0	1	2	1	0
Output value	−4	−2	0	2	4

7.

Input value	10	7	4	7	10
Output value	3	6	9	12	15

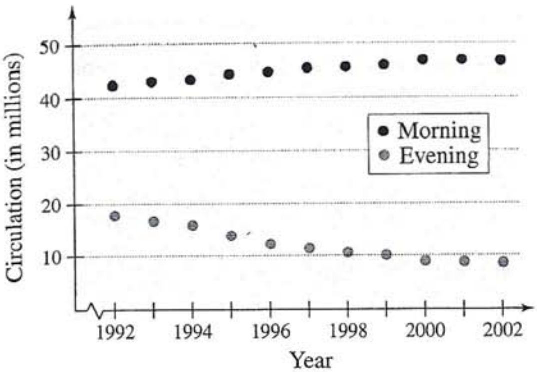
8.

Input value	0	3	9	12	15
Output value	3	3	3	3	3

In Exercises 9 and 10, which sets of ordered pairs represent functions from A to B? Explain.

9. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$
   (a) $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$
   (b) $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$
   (c) $\{(0, 0), (1, 0), (2, 0), (3, 0)\}$
   (d) $\{(0, 2), (3, 0), (1, 1)\}$
10. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$
   (a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$
   (b) $\{(a, 1), (b, 2), (c, 3)\}$
   (c) $\{(1, a), (0, a), (2, c), (3, b)\}$
   (d) $\{(c, 0), (b, 0), (a, 3)\}$

Circulation of Newspapers **In Exercises 11 and 12, use the graph, which shows the circulation (in millions) of daily newspapers in the United States.** (Source: Editor & Publisher Company)



11. Is the circulation of morning newspapers a function of the year? Is the circulation of evening newspapers a function of the year? Explain.

12. Let $f(x)$ represent the circulation of evening newspapers in year x. Find $f(1998)$.

In Exercises 13–24, determine whether the equation represents *y* as a function of *x*.

13. $x^2 + y^2 = 4$ **14.** $x = y^2$

15. $x^2 + y = 4$ **16.** $x + y^2 = 4$

17. $2x + 3y = 4$ **18.** $(x - 2)^2 + y^2 = 4$

19. $y^2 = x^2 - 1$ **20.** $y = \sqrt{x + 5}$

21. $y = |4 - x|$ **22.** $|y| = 4 - x$

23. $x = 14$ **24.** $y = -75$

In Exercises 25–38, evaluate the function at each specified value of the independent variable and simplify.

25. $f(x) = 2x - 3$
(a) $f(1)$ (b) $f(-3)$ (c) $f(x - 1)$

26. $g(y) = 7 - 3y$
(a) $g(0)$ (b) $g\left(\frac{7}{3}\right)$ (c) $g(s + 2)$

27. $V(r) = \frac{4}{3}\pi r^3$
(a) $V(3)$ (b) $V\left(\frac{3}{2}\right)$ (c) $V(2r)$

28. $h(t) = t^2 - 2t$
(a) $h(2)$ (b) $h(1.5)$ (c) $h(x + 2)$

29. $f(y) = 3 - \sqrt{y}$
(a) $f(4)$ (b) $f(0.25)$ (c) $f(4x^2)$

30. $f(x) = \sqrt{x + 8} + 2$
(a) $f(-8)$ (b) $f(1)$ (c) $f(x - 8)$

31. $q(x) = \dfrac{1}{x^2 - 9}$
(a) $q(0)$ (b) $q(3)$ (c) $q(y + 3)$

32. $q(t) = \dfrac{2t^2 + 3}{t^2}$
(a) $q(2)$ (b) $q(0)$ (c) $q(-x)$

33. $f(x) = \dfrac{|x|}{x}$
(a) $f(2)$ (b) $f(-2)$ (c) $f(x - 1)$

34. $f(x) = |x| + 4$
(a) $f(2)$ (b) $f(-2)$ (c) $f(x^2)$

35. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$
(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$

36. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$
(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

37. $f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$
(a) $f(-2)$ (b) $f\left(-\frac{1}{2}\right)$ (c) $f(3)$

38. $f(x) = \begin{cases} 4 - 5x, & x \le -2 \\ 0, & -2 < x < 2 \\ x^2 + 1, & x > 2 \end{cases}$
(a) $f(-3)$ (b) $f(4)$ (c) $f(-1)$

In Exercises 39–44, complete the table.

39. $f(x) = x^2 - 3$

x	-2	-1	0	1	2
$f(x)$					

40. $g(x) = \sqrt{x - 3}$

x	3	4	5	6	7
$g(x)$					

41. $h(t) = \frac{1}{2}|t + 3|$

t	-5	-4	-3	-2	-1
$h(t)$					

42. $f(s) = \dfrac{|s - 2|}{s - 2}$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

43. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x-2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2
$f(x)$					

44. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

x	1	2	3	4	5
$f(x)$					

In Exercises 45–52, find all real values of x such that $f(x) = 0$.

45. $f(x) = 15 - 3x$
46. $f(x) = 5x + 1$
47. $f(x) = \dfrac{3x - 4}{5}$
48. $f(x) = \dfrac{12 - x^2}{5}$
49. $f(x) = x^2 - 9$
50. $f(x) = x^2 - 8x + 15$
51. $f(x) = x^3 - x$
52. $f(x) = x^3 - x^2 - 4x + 4$

In Exercises 53–56, find the value(s) of x for which $f(x) = g(x)$.

53. $f(x) = x^2 + 2x + 1, \quad g(x) = 3x + 3$
54. $f(x) = x^4 - 2x^2, \quad g(x) = 2x^2$
55. $f(x) = \sqrt{3x} + 1, \quad g(x) = x + 1$
56. $f(x) = \sqrt{x} - 4, \quad g(x) = 2 - x$

In Exercises 57–70, find the domain of the function.

57. $f(x) = 5x^2 + 2x - 1$
58. $g(x) = 1 - 2x^2$
59. $h(t) = \dfrac{4}{t}$
60. $s(y) = \dfrac{3y}{y + 5}$
61. $g(y) = \sqrt{y - 10}$
62. $f(t) = \sqrt[3]{t + 4}$
63. $f(x) = \sqrt[4]{1 - x^2}$
64. $f(x) = \sqrt[4]{x^2 + 3x}$
65. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$
66. $h(x) = \dfrac{10}{x^2 - 2x}$
67. $f(s) = \dfrac{\sqrt{s - 1}}{s - 4}$
68. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$
69. $f(x) = \dfrac{x - 4}{\sqrt{x}}$
70. $f(x) = \dfrac{x - 5}{\sqrt{x^2 - 9}}$

In Exercises 71–74, assume that the domain of f is the set $A = \{-2, -1, 0, 1, 2\}$. Determine the set of ordered pairs that represents the function f.

71. $f(x) = x^2$
72. $f(x) = x^2 - 3$
73. $f(x) = |x| + 2$
74. $f(x) = |x + 1|$

Exploration In Exercises 75–78, match the data with one of the following functions

$f(x) = cx$, $g(x) = cx^2$, $h(x) = c\sqrt{|x|}$, and $r(x) = \dfrac{c}{x}$

and determine the value of the constant c that will make the function fit the data in the table.

75.

x	-4	-1	0	1	4
y	-32	-2	0	-2	-32

76.

x	-4	-1	0	1	4
y	-1	$-\frac{1}{4}$	0	$\frac{1}{4}$	1

77.

x	-4	-1	0	1	4
y	-8	-32	Undef.	32	8

78.

x	-4	-1	0	1	4
y	6	3	0	3	6

In Exercises 79–86, find the difference quotient and simplify your answer.

79. $f(x) = x^2 - x + 1, \quad \dfrac{f(2 + h) - f(2)}{h}, h \neq 0$

80. $f(x) = 5x - x^2, \quad \dfrac{f(5 + h) - f(5)}{h}, h \neq 0$

81. $f(x) = x^3 + 3x, \quad \dfrac{f(x + h) - f(x)}{h}, h \neq 0$

82. $f(x) = 4x^2 - 2x, \quad \dfrac{f(x + h) - f(x)}{h}, h \neq 0$

83. $g(x) = \dfrac{1}{x^2}, \quad \dfrac{g(x) - g(3)}{x - 3}, x \neq 3$

84. $f(t) = \dfrac{1}{t - 2}, \quad \dfrac{f(t) - f(1)}{t - 1}, t \neq 1$

85. $f(x) = \sqrt{5x}, \quad \dfrac{f(x) - f(5)}{x - 5}, x \neq 5$

86. $f(x) = x^{2/3} + 1, \quad \dfrac{f(x) - f(8)}{x - 8}, x \neq 8$

87. ***Geometry*** Write the area A of a square as a function of its perimeter P.

88. ***Geometry*** Write the area A of a circle as a function of its circumference C.

The symbol $\int$ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

1.5 Exercises

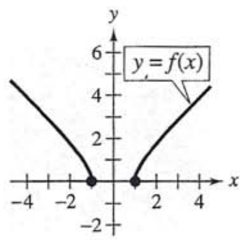
VOCABULARY CHECK: Fill in the blanks.

1. The graph of a function f is the collection of ________ ________ or $(x, f(x))$ such that x is in the domain of f.
2. The ________ ________ ________ is used to determine whether the graph of an equation is a function of y in terms of x.
3. The ________ of a function f are the values of x for which $f(x) = 0$.
4. A function f is ________ on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
5. A function value $f(a)$ is a relative ________ of f if there exists an interval (x_1, x_2) containing a such that $x_1 < x < x_2$ implies $f(a) \geq f(x)$.
6. The ________ ________ ________ ________ between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points, and this line is called the ________ line.
7. A function f is ________ if for the each x in the domain of f, $f(-x) = -f(x)$.
8. A function f is ________ if its graph is symmetric with respect to the y-axis.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, use the graph of the function to find the domain and range of f.

1.



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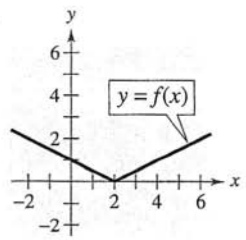

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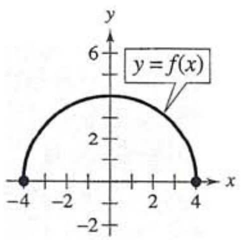

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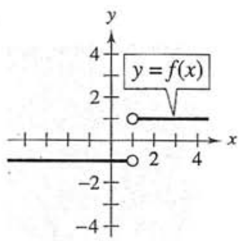
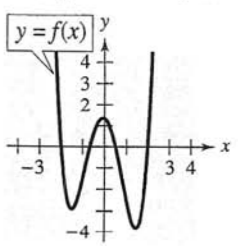
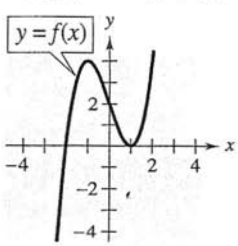


In Exercises 5–8, use the graph of the function to find the indicated function values.

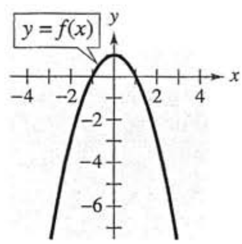
5. (a) $f(-2)$ (b) $f(-1)$
(c) $f\left(\frac{1}{2}\right)$ (d) $f(1)$



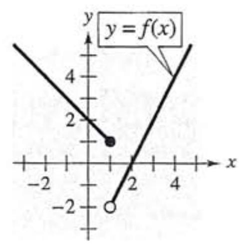
6. (a) $f(-1)$ (b) $f(2)$
(c) $f(0)$ (d) $f(1)$



7. (a) $f(-2)$ (b) $f(1)$
(c) $f(0)$ (d) $f(2)$

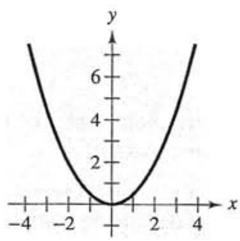


8. (a) $f(2)$ (b) $f(1)$
(c) $f(3)$ (d) $f(-1)$

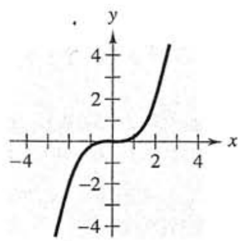


In Exercises 9–14, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

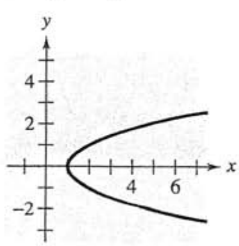
9. $y = \frac{1}{2}x^2$



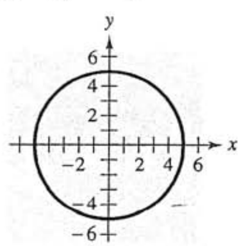
10. $y = \frac{1}{4}x^3$



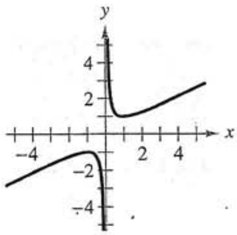
11. $x - y^2 = 1$



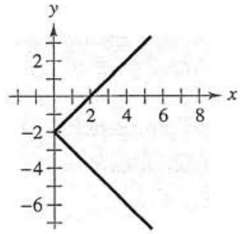
12. $x^2 + y^2 = 25$



13. $x^2 = 2xy - 1$



14. $x = |y + 2|$



In Exercises 15–24, find the zeros of the function algebraically.

15. $f(x) = 2x^2 - 7x - 30$

16. $f(x) = 3x^2 + 22x - 16$

17. $f(x) = \dfrac{x}{9x^2 - 4}$

18. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$

19. $f(x) = \frac{1}{2}x^3 - x$

20. $f(x) = x^3 - 4x^2 - 9x + 36$

21. $f(x) = 4x^3 - 24x^2 - x + 6$

22. $f(x) = 9x^4 - 25x^2$

23. $f(x) = \sqrt{2x} - 1$

24. $f(x) = \sqrt{3x + 2}$

In Exercises 25–30, (a) use a graphing utility to graph the function and find the zeros of the function and (b) verify your results from part (a) algebraically.

25. $f(x) = 3 + \dfrac{5}{x}$

26. $f(x) = x(x - 7)$

27. $f(x) = \sqrt{2x + 11}$

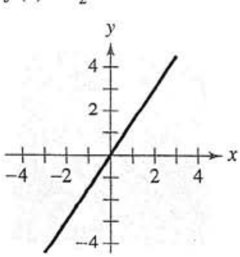
28. $f(x) = \sqrt{3x - 14} - 8$

29. $f(x) = \dfrac{3x - 1}{x - 6}$

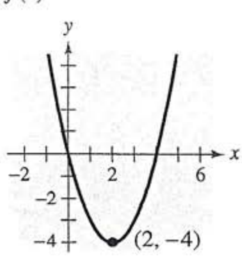
30. $f(x) = \dfrac{2x^2 - 9}{3 - x}$

In Exercises 31–38, determine the intervals over which the function is increasing, decreasing, or constant.

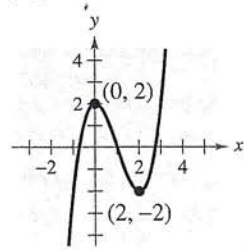
31. $f(x) = \frac{3}{2}x$



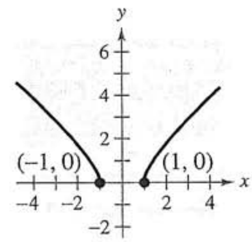
32. $f(x) = x^2 - 4x$



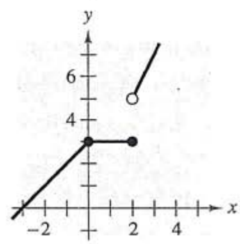
33. $f(x) = x^3 - 3x^2 + 2$



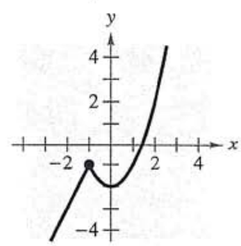
34. $f(x) = \sqrt{x^2 - 1}$



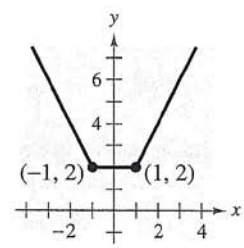
35. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$



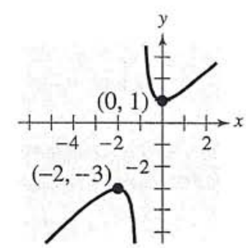
36. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$



37. $f(x) = |x + 1| + |x - 1|$



38. $f(x) = \dfrac{x^2 + x + 1}{x + 1}$



In Exercises 39–48, (a) use a graphing utility to graph the function and visually determine the intervals over which the function is increasing, decreasing, or constant, and (b) make a table of values to verify whether the function is increasing, decreasing, or constant over the intervals you identified in part (a).

39. $f(x) = 3$
40. $g(x) = x$
41. $g(s) = \dfrac{s^2}{4}$
42. $h(x) = x^2 - 4$
43. $f(t) = -t^4$
44. $f(x) = 3x^4 - 6x^2$
45. $f(x) = \sqrt{1-x}$
46. $f(x) = x\sqrt{x+3}$
47. $f(x) = x^{3/2}$
48. $f(x) = x^{2/3}$

In Exercises 49–54, use a graphing utility to graph the function and approximate (to two decimal places) any relative minimum or relative maximum values.

49. $f(x) = (x-4)(x+2)$
50. $f(x) = 3x^2 - 2x - 5$
51. $f(x) = -x^2 + 3x - 2$
52. $f(x) = -2x^2 + 9x$
53. $f(x) = x(x-2)(x+3)$
54. $f(x) = x^3 - 3x^2 - x + 1$

In Exercises 55–62, graph the function and determine the interval(s) for which $f(x) \geq 0$.

55. $f(x) = 4 - x$
56. $f(x) = 4x + 2$
57. $f(x) = x^2 + x$
58. $f(x) = x^2 - 4x$
59. $f(x) = \sqrt{x-1}$
60. $f(x) = \sqrt{x+2}$
61. $f(x) = -(1 + |x|)$
62. $f(x) = \frac{1}{2}(2 + |x|)$

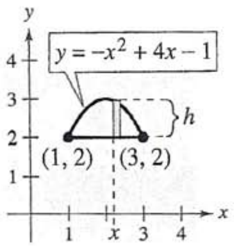
In Exercises 63–70, find the average rate of change of the function from x_1 to x_2.

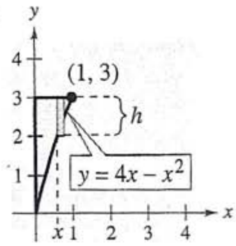
	Function	*x-Values*
63.	$f(x) = -2x + 15$	$x_1 = 0, x_2 = 3$
64.	$f(x) = 3x + 8$	$x_1 = 0, x_2 = 3$
65.	$f(x) = x^2 + 12x - 4$	$x_1 = 1, x_2 = 5$
66.	$f(x) = x^2 - 2x + 8$	$x_1 = 1, x_2 = 5$
67.	$f(x) = x^3 - 3x^2 - x$	$x_1 = 1, x_2 = 3$
68.	$f(x) = -x^3 + 6x^2 + x$	$x_1 = 1, x_2 = 6$
69.	$f(x) = -\sqrt{x-2} + 5$	$x_1 = 3, x_2 = 11$
70.	$f(x) = -\sqrt{x+1} + 3$	$x_1 = 3, x_2 = 8$

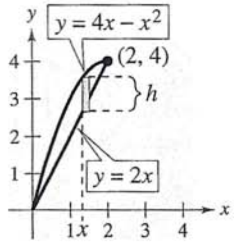
In Exercises 71–76, determine whether the function is even, odd, or neither. Then describe the symmetry.

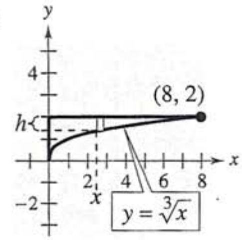
71. $f(x) = x^6 - 2x^2 + 3$
72. $h(x) = x^3 - 5$
73. $g(x) = x^3 - 5x$
74. $f(x) = x\sqrt{1-x^2}$
75. $f(t) = t^2 + 2t - 3$
76. $g(s) = 4s^{2/3}$

In Exercises 77–80, write the height h of the rectangle as a function of x.

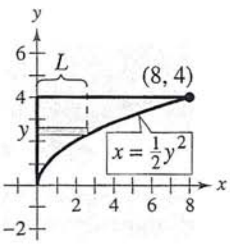
77.


78.


79.


80.


In Exercises 81–84, write the length L of the rectangle as a function of y.

81.


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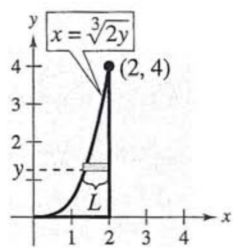
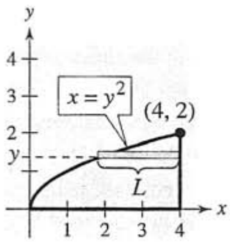


83.


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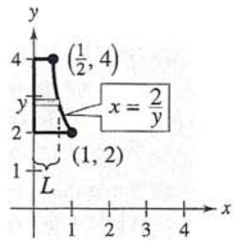


85. *Electronics* The number of lumens (time rate of flow of light) L from a fluorescent lamp can be approximated by the model

$$L = -0.294x^2 + 97.744x - 664.875, \quad 20 \leq x \leq 90$$

where x is the wattage of the lamp.

(a) Use a graphing utility to graph the function.

(b) Use the graph from part (a) to estimate the wattage necessary to obtain 2000 lumens.

1.6 Exercises

VOCABULARY CHECK: Match each function with its name.

1. $f(x) = [\![x]\!]$
2. $f(x) = x$
3. $f(x) = \dfrac{1}{x}$
4. $f(x) = x^2$
5. $f(x) = \sqrt{x}$
6. $f(x) = c$
7. $f(x) = |x|$
8. $f(x) = x^3$
9. $f(x) = ax + b$

(a) squaring function (b) square root function (c) cubic function

(d) linear function (e) constant function (f) absolute value function

(e) greatest integer function (h) reciprocal function (i) identity function

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, (a) write the linear function f such that it has the indicated function values and (b) sketch the graph of the function.

1. $f(1) = 4, f(0) = 6$
2. $f(-3) = -8, f(1) = 2$
3. $f(5) = -4, f(-2) = 17$
4. $f(3) = 9, f(-1) = -11$
5. $f(-5) = -1, f(5) = -1$
6. $f(-10) = 12, f(16) = -1$
7. $f\left(\frac{1}{2}\right) = -6, f(4) = -3$
8. $f\left(\frac{2}{3}\right) = -\frac{15}{2}, f(-4) = -11$

In Exercises 9–28, use a graphing utility to graph the function. Be sure to choose an appropriate viewing window.

9. $f(x) = -x - \frac{3}{4}$
10. $f(x) = 3x - \frac{5}{2}$
11. $f(x) = -\frac{1}{6}x - \frac{5}{2}$
12. $f(x) = \frac{5}{6} - \frac{2}{3}x$
13. $f(x) = x^2 - 2x$
14. $f(x) = -x^2 + 8x$
15. $h(x) = -x^2 + 4x + 12$
16. $g(x) = x^2 - 6x - 16$
17. $f(x) = x^3 - 1$
18. $f(x) = 8 - x^3$
19. $f(x) = (x - 1)^3 + 2$
20. $g(x) = 2(x + 3)^3 + 1$
21. $f(x) = 4\sqrt{x}$
22. $f(x) = 4 - 2\sqrt{x}$
23. $g(x) = 2 - \sqrt{x + 4}$
24. $h(x) = \sqrt{x + 2} + 3$
25. $f(x) = -\dfrac{1}{x}$
26. $f(x) = 4 + \dfrac{1}{x}$
27. $h(x) = \dfrac{1}{x + 2}$
28. $k(x) = \dfrac{1}{x - 3}$

In Exercises 29–36, evaluate the function for the indicated values.

29. $f(x) = [\![x]\!]$
    (a) $f(2.1)$ (b) $f(2.9)$ (c) $f(-3.1)$ (d) $f\left(\frac{7}{2}\right)$
30. $g(x) = 2[\![x]\!]$
    (a) $g(-3)$ (b) $g(0.25)$ (c) $g(9.5)$ (d) $g\left(\frac{11}{3}\right)$
31. $h(x) = [\![x + 3]\!]$
    (a) $h(-2)$ (b) $h\left(\frac{1}{2}\right)$ (c) $h(4.2)$ (d) $h(-21.6)$
32. $f(x) = 4[\![x]\!] + 7$
    (a) $f(0)$ (b) $f(-1.5)$ (c) $f(6)$ (d) $f\left(\frac{5}{3}\right)$
33. $h(x) = [\![3x - 1]\!]$
    (a) $h(2.5)$ (b) $h(-3.2)$ (c) $h\left(\frac{7}{3}\right)$ (d) $h\left(-\frac{21}{3}\right)$
34. $k(x) = [\![\frac{1}{2}x + 6]\!]$
    (a) $k(5)$ (b) $k(-6.1)$ (c) $k(0.1)$ (d) $k(15)$
35. $g(x) = 3[\![x - 2]\!] + 5$
    (a) $g(-2.7)$ (b) $g(-1)$ (c) $g(0.8)$ (d) $g(14.5)$
36. $g(x) = -7[\![x + 4]\!] + 6$
    (a) $g\left(\frac{1}{8}\right)$ (b) $g(9)$ (c) $g(-4)$ (d) $g\left(\frac{3}{2}\right)$

In Exercises 37–42, sketch the graph of the function.

37. $g(x) = -[\![x]\!]$
38. $g(x) = 4[\![x]\!]$
39. $g(x) = [\![x]\!] - 2$
40. $g(x) = [\![x]\!] - 1$
41. $g(x) = [\![x + 1]\!]$
42. $g(x) = [\![x - 3]\!]$

In Exercises 43–50, graph the function.

43. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \ge 0 \end{cases}$
44. $g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$
45. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \ge 0 \end{cases}$
46. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \le 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$
47. $f(x) = \begin{cases} x^2 + 5, & x \le 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases}$

48. $h(x) = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 2, & x \geq 0 \end{cases}$

49. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

50. $k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$

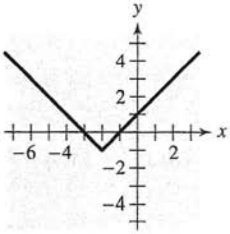
In Exercises 51 and 52, (a) use a graphing utility to graph the function, (b) state the domain and range of the function, and (c) describe the pattern of the graph.

51. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

52. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$

In Exercises 53–60, (a) identify the parent function and the transformed parent function shown in the graph, (b) write an equation for the function shown in the graph, and (c) use a graphing utility to verify your answers in parts (a) and (b).

53.



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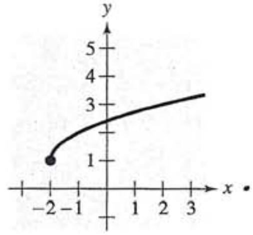
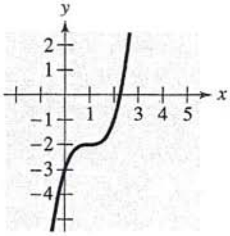


55.



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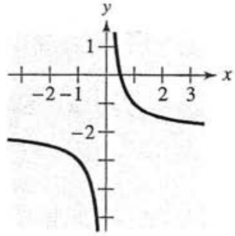

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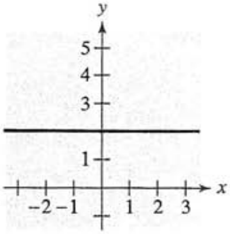

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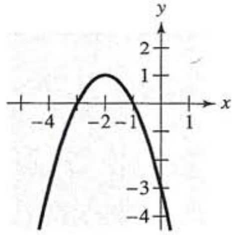

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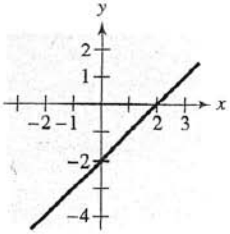

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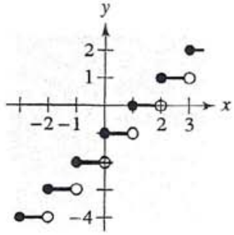


61. *Communications* The cost of a telephone call between Denver and Boise is \$0.60 for the first minute and \$0.42 for each additional minute or portion of a minute. A model for the total cost C (in dollars) of the phone call is $C = 0.60 - 0.42[\![1 - t]\!]$, $t > 0$ where t is the length of the phone call in minutes.

(a) Sketch the graph of the model.

(b) Determine the cost of a call lasting 12 minutes and 30 seconds.

62. *Communications* The cost of using a telephone calling card is \$1.05 for the first minute and \$0.38 for each additional minute or portion of a minute.

(a) A customer needs a model for the cost C of using a calling card for a call lasting t minutes. Which of the following is the appropriate model? Explain.

$C_1(t) = 1.05 + 0.38[\![t - 1]\!]$

$C_2(t) = 1.05 - 0.38[\![-(t - 1)]\!]$

(b) Graph the appropriate model. Determine the cost of a call lasting 18 minutes and 45 seconds.

63. *Delivery Charges* The cost of sending an overnight package from Los Angeles to Miami is \$10.75 for a package weighing up to but not including 1 pound and \$3.95 for each additional pound or portion of a pound. A model for the total cost C (in dollars) of sending the package is $C = 10.75 + 3.95[\![x]\!]$, $x > 0$ where x is the weight in pounds.

(a) Sketch a graph of the model.

(b) Determine the cost of sending a package that weighs 10.33 pounds.

64. *Delivery Charges* The cost of sending an overnight package from New York to Atlanta is \$9.80 for a package weighing up to but not including 1 pound and \$2.50 for each additional pound or portion of a pound.

(a) Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds, $x > 0$.

(b) Sketch the graph of the function.

65. *Wages* A mechanic is paid \$12.00 per hour for regular time and time-and-a-half for overtime. The weekly wage function is given by

$$W(h) = \begin{cases} 12h, & 0 < h \leq 40 \\ 18(h - 40) + 480, & h > 40 \end{cases}$$

where h is the number of hours worked in a week.

(a) Evaluate $W(30)$, $W(40)$, $W(45)$, and $W(50)$.

(b) The company increased the regular work week to 45 hours. What is the new weekly wage function?

66. ***Snowstorm*** During a nine-hour snowstorm, it snows at a rate of 1 inch per hour for the first 2 hours, at a rate of 2 inches per hour for the next 6 hours, and at a rate of 0.5 inch per hour for the final hour. Write and graph a piecewise-defined function that gives the depth of the snow during the snowstorm. How many inches of snow accumulated from the storm?

Model It

67. ***Revenue*** The table shows the monthly revenue y (in thousands of dollars) of a landscaping business for each month of the year 2005, with $x = 1$ representing January.

Month, x	Revenue, y
1	5.2
2	5.6
3	6.6
4	8.3
5	11.5
6	15.8
7	12.8
8	10.1
9	8.6
10	6.9
11	4.5
12	2.7

A mathematical model that represents these data is

$$f(x) = \begin{cases} -1.97x + 26.3 \\ 0.505x^2 - 1.47x + 6.3 \end{cases}$$

(a) What is the domain of each part of the piecewise-defined function? How can you tell? Explain your reasoning.

(b) Sketch a graph of the model.

(c) Find $f(5)$ and $f(11)$, and interpret your results in the context of the problem.

(d) How do the values obtained from the model in part (b) compare with the actual data values?

68. ***Fluid Flow*** The intake pipe of a 100-gallon tank has a flow rate of 10 gallons per minute, and two drainpipes have flow rates of 5 gallons per minute each. The figure shows the volume V of fluid in the tank as a function of time t. Determine the combination of the input pipe and drain pipes in which the fluid is flowing in specific subintervals of the 1 hour of time shown on the graph. (There are many correct answers.)

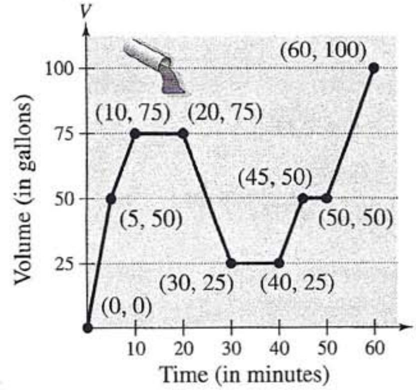


FIGURE FOR 68

Synthesis

True or False? **In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.**

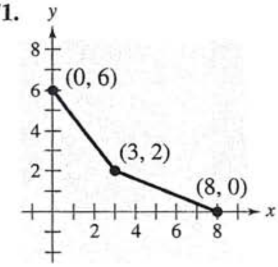
69. A piecewise-defined function will always have at least one x-intercept or at least one y-intercept.

70. $f(x) = \begin{cases} 2, & 1 \le x < 2 \\ 4, & 2 \le x < 3 \\ 6, & 3 \le x < 4 \end{cases}$

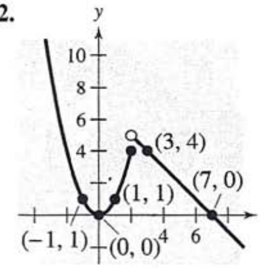
can be rewritten as $f(x) = 2[\![x]\!], \quad 1 \le x < 4$.

Exploration **In Exercises 71 and 72, write equations for the piecewise-defined function shown in the graph.**

71.



72.



Skills Review

In Exercises 73 and 74, solve the inequality and sketch the solution on the real number line.

73. $3x + 4 \le 12 - 5x$

74. $2x + 1 > 6x - 9$

In Exercises 75 and 76, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.

75. L_1: $(-2, -2), (2, 10)$
L_2: $(-1, 3), (3, 9)$

76. L_1: $(-1, -7), (4, 3)$
L_2: $(1, 5), (-2, -7)$

1.7 Exercises

VOCABULARY CHECK:

In Exercises 1–5, fill in the blanks.

1. Horizontal shifts, vertical shifts, and reflections are called ________ transformations.
2. A reflection in the x-axis of $y = f(x)$ is represented by $h(x) =$ ________, while a reflection in the y-axis of $y = f(x)$ is represented by $h(x) =$ ________.
3. Transformations that cause a distortion in the shape of the graph of $y = f(x)$ are called ________ transformations.
4. A nonrigid transformation of $y = f(x)$ represented by $h(x) = f(cx)$ is a ________ ________ if $c > 1$ and a ________ ________ if $0 < c < 1$.
5. A nonrigid transformation of $y = f(x)$ represented by $g(x) = cf(x)$ is a ________ ________ if $c > 1$ and a ________ ________ if $0 < c < 1$.
6. Match the rigid transformation of $y = f(x)$ with the correct representation of the graph of h, where $c > 0$.

(a) $h(x) = f(x) + c$	(i) A horizontal shift of f, c units to the right
(b) $h(x) = f(x) - c$	(ii) A vertical shift of f, c units downward
(c) $h(x) = f(x + c)$	(iii) A horizontal shift of f, c units to the left
(d) $h(x) = f(x - c)$	(iv) A vertical shift of f, c units upward

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

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1. For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -1, 1$, and 3.
   (a) $f(x) = |x| + c$
   (b) $f(x) = |x - c|$
   (c) $f(x) = |x + 4| + c$
2. For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3.
   (a) $f(x) = \sqrt{x} + c$
   (b) $f(x) = \sqrt{x - c}$
   (c) $f(x) = \sqrt{x - 3} + c$
3. For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -2, 0$, and 2.
   (a) $f(x) = [\![x]\!] + c$
   (b) $f(x) = [\![x + c]\!]$
   (c) $f(x) = [\![x - 1]\!] + c$
4. For each function, sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3.
   (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \ge 0 \end{cases}$
   (b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \ge 0 \end{cases}$

In Exercises 5–8, use the graph of f to sketch each graph. To print an enlarged copy of the graph go to the website *www.mathgraphs.com.*

5. (a) $y = f(x) + 2$
   (b) $y = f(x - 2)$
   (c) $y = 2f(x)$
   (d) $y = -f(x)$
   (e) $y = f(x + 3)$
   (f) $y = f(-x)$
   (g) $y = f\left(\frac{1}{2}x\right)$

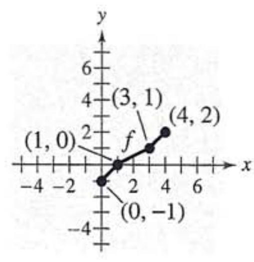


FIGURE FOR 5

6. (a) $y = f(-x)$
   (b) $y = f(x) + 4$
   (c) $y = 2f(x)$
   (d) $y = -f(x - 4)$
   (e) $y = f(x) - 3$
   (f) $y = -f(x) - 1$
   (g) $y = f(2x)$

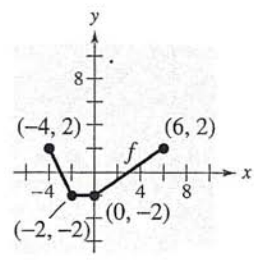


FIGURE FOR 6

7. (a) $y = f(x) - 1$
   (b) $y = f(x - 1)$
   (c) $y = f(-x)$
   (d) $y = f(x + 1)$
   (e) $y = -f(x - 2)$
   (f) $y = \frac{1}{2}f(x)$
   (g) $y = f(2x)$

8. (a) $y = f(x - 5)$
   (b) $y = -f(x) + 3$
   (c) $y = \frac{1}{3}f(x)$
   (d) $y = -f(x + 1)$
   (e) $y = f(-x)$
   (f) $y = f(x) - 10$
   (g) $y = f\left(\frac{1}{3}x\right)$

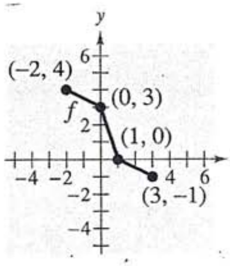


FIGURE FOR 7

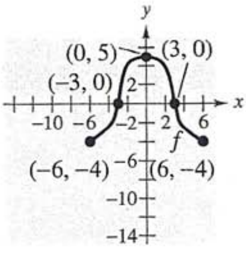
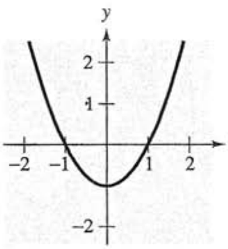


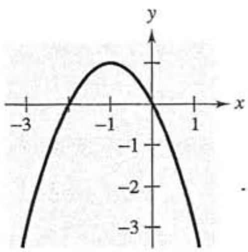
FIGURE FOR 8

9. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.

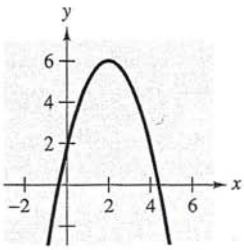
(a)



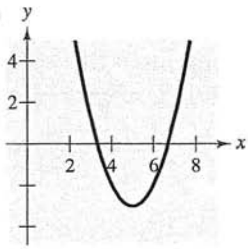
(b)



(c)

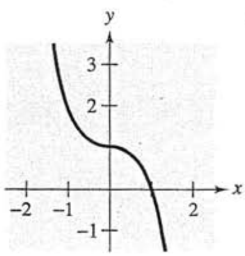


(d)

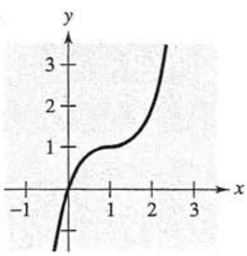


10. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.

(a)



(b)



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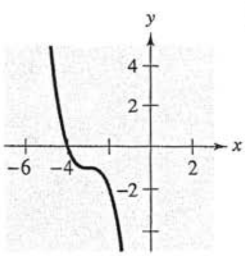

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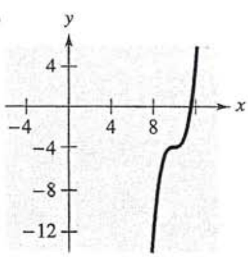
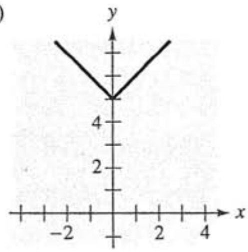
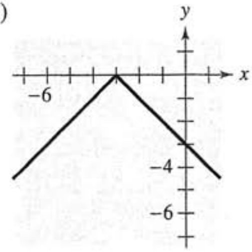


11. Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown.

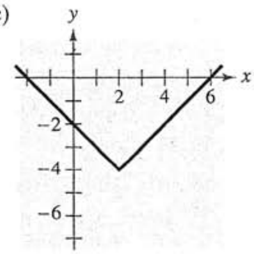
(a)



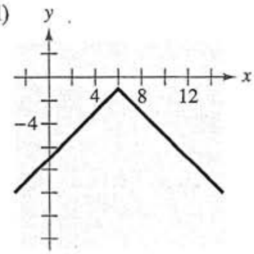
(b)



(c)

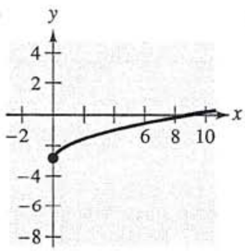


(d)

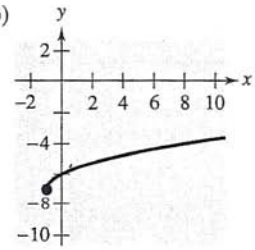


12. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

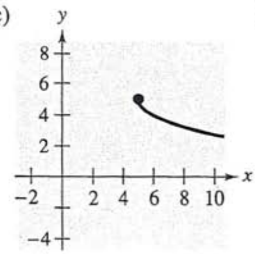
(a)



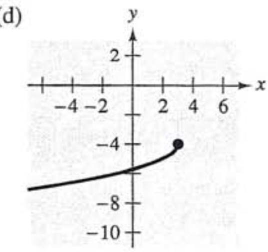
(b)



(c)

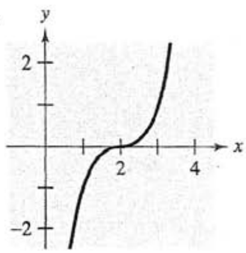


(d)



In Exercises 13–18, identify the parent function and the transformation shown in the graph. Write an equation for the function shown in the graph.

13.



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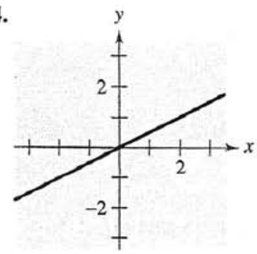
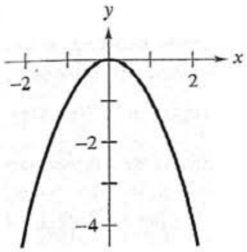
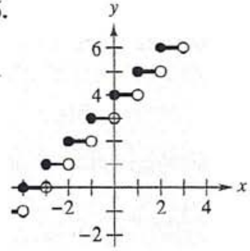
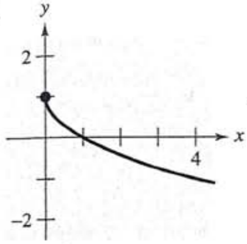
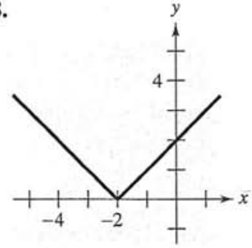


15. 

16. 

17. 

18. 

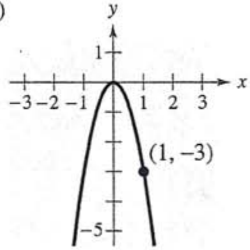
In Exercises 19–42, *g* is related to one of the parent functions described in this chapter. (a) Identify the parent function *f*. (b) Describe the sequence of tranformations from *f* to *g*. (c) Sketch the graph of *g*. (d) Use function notation to write *g* in terms of *f*.

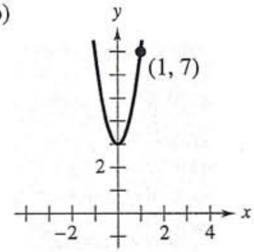
19. $g(x) = 12 - x^2$
20. $g(x) = (x - 8)^2$
21. $g(x) = x^3 + 7$
22. $g(x) = -x^3 - 1$
23. $g(x) = \frac{2}{3}x^2 + 4$
24. $g(x) = 2(x - 7)^2$
25. $g(x) = 2 - (x + 5)^2$
26. $g(x) = -(x + 10)^2 + 5$
27. $g(x) = \sqrt{3x}$
28. $g(x) = \sqrt{\frac{1}{4}x}$
29. $g(x) = (x - 1)^3 + 2$
30. $g(x) = (x + 3)^3 - 10$
31. $g(x) = -|x| - 2$
32. $g(x) = 6 - |x + 5|$
33. $g(x) = -|x + 4| + 8$
34. $g(x) = |-x + 3| + 9$
35. $g(x) = 3 - [\![x]\!]$
36. $g(x) = 2[\![x + 5]\!]$
37. $g(x) = \sqrt{x - 9}$
38. $g(x) = \sqrt{x + 4} + 8$
39. $g(x) = \sqrt{7 - x} - 2$
40. $g(x) = -\sqrt{x + 1} - 6$
41. $g(x) = \sqrt{\frac{1}{2}x} - 4$
42. $g(x) = \sqrt{3x} + 1$

In Exercises 43–50, write an equation for the function that is described by the given characteristics.

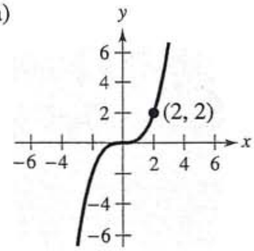
43. The shape of $f(x) = x^2$, but moved two units to the right and eight units downward
44. The shape of $f(x) = x^2$, but moved three units to the left, seven units upward, and reflected in the x-axis
45. The shape of $f(x) = x^3$, but moved 13 units to the right
46. The shape of $f(x) = x^3$, but moved six units to the left, six units downward, and reflected in the y-axis
47. The shape of $f(x) = |x|$, but moved 10 units upward and reflected in the x-axis
48. The shape of $f(x) = |x|$, but moved one unit to the left and seven units downward
49. The shape of $f(x) = \sqrt{x}$, but moved six units to the left and reflected in both the x-axis and the y-axis
50. The shape of $f(x) = \sqrt{x}$, but moved nine units downward and reflected in both the x-axis and the y-axis

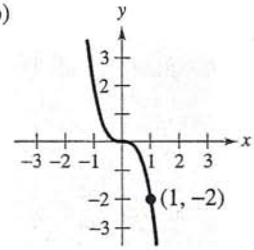
51. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown.

(a) 

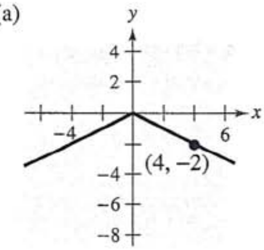
(b) 

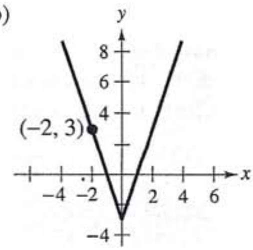
52. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.

(a) 

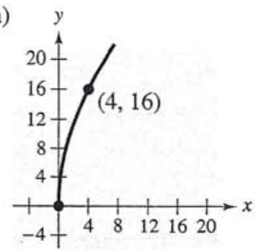
(b) 

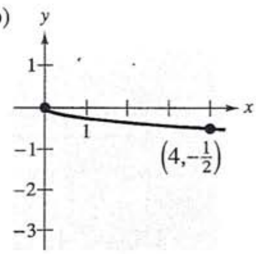
53. Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown.

(a) 

(b) 

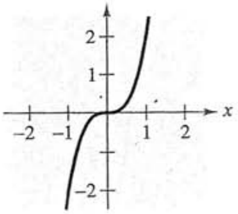
54. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

(a) 

(b) 

In Exercises 55–60, identify the parent function and the transformation shown in the graph. Write an equation for the function shown in the graph. Then use a graphing utility to verify your answer.

55.



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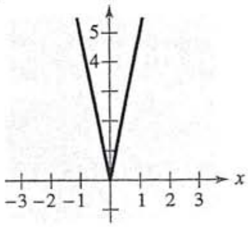

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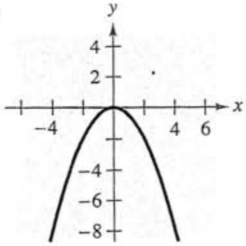

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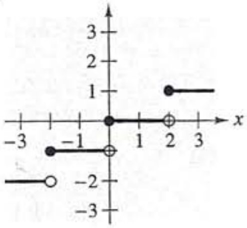

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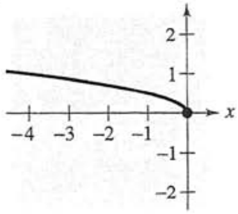

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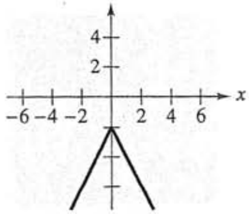
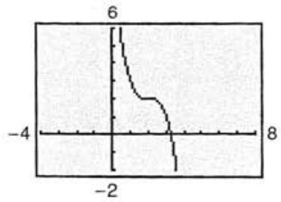


Graphical Analysis **In Exercises 61–64, use the viewing window shown to write a possible equation for the transformation of the parent function.**

61.



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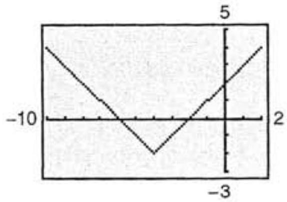

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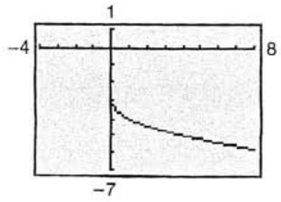

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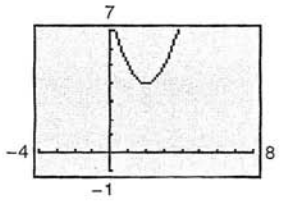
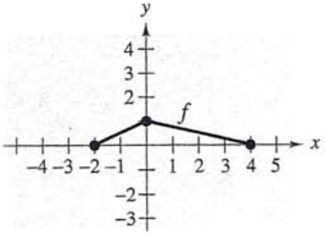


Graphical Reasoning **In Exercises 65 and 66, use the graph of f to sketch the graph of g. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

65.

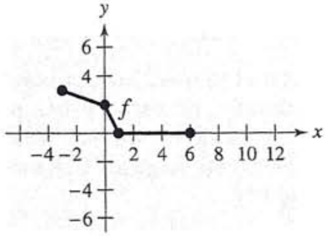


(a) $g(x) = f(x) + 2$ (b) $g(x) = f(x) - 1$

(c) $g(x) = f(-x)$ (d) $g(x) = -2f(x)$

(e) $g(x) = f(4x)$ (f) $g(x) = f\left(\frac{1}{2}x\right)$

66.



(a) $g(x) = f(x) - 5$ (b) $g(x) = f(x) + \frac{1}{2}$

(c) $g(x) = f(-x)$ (d) $g(x) = -4f(x)$

(e) $g(x) = f(2x) + 1$ (f) $g(x) = f\left(\frac{1}{4}x\right) - 2$

Model It

67. ***Fuel Use*** The amounts of fuel F (in billions of gallons) used by trucks from 1980 through 2002 can be approximated by the function

$$F = f(t) = 20.6 + 0.035t^2, \quad 0 \le t \le 22$$

where t represents the year, with $t = 0$ corresponding to 1980. (Source: U.S. Federal Highway Administration)

(a) Describe the transformation of the parent function $f(x) = x^2$. Then sketch the graph over the specified domain.

(b) Find the average rate of change of the function from 1980 to 2002. Interpret your answer in the context of the problem.

(c) Rewrite the function so that $t = 0$ represents 1990. Explain how you got your answer.

(d) Use the model from part (c) to predict the amount of fuel used by trucks in 2010. Does your answer seem reasonable? Explain.

68. ***Finance*** The amounts M (in trillions of dollars) of mortgage debt outstanding in the United States from 1990 through 2002 can be approximated by the function

$$M = f(t) = 0.0054(t + 20.396)^2, \quad 0 \le t \le 12$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: Board of Governors of the Federal Reserve System)

(a) Describe the transformation of the parent function $f(x) = x^2$. Then sketch the graph over the specified domain.

(b) Rewrite the function so that $t = 0$ represents 2000. Explain how you got your answer.

Synthesis

True or False? **In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.**

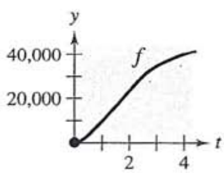
69. The graphs of

$$f(x) = |x| + 6 \quad \text{and} \quad f(x) = |-x| + 6$$

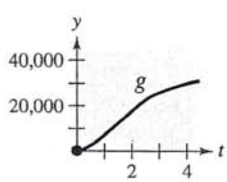
are identical.

70. If the graph of the parent function $f(x) = x^2$ is moved six units to the right, three units upward, and reflected in the x-axis, then the point $(-2, 19)$ will lie on the graph of the transformation.

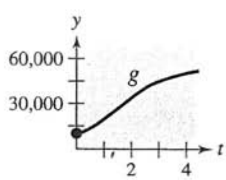
71. ***Describing Profits*** Management originally predicted that the profits from the sales of a new product would be approximated by the graph of the function f shown. The actual profits are shown by the function g along with a verbal description. Use the concepts of transformations of graphs to write g in terms of f.



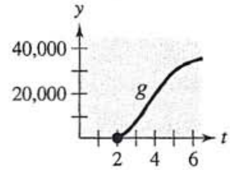
(a) The profits were only three-fourths as large as expected.



(b) The profits were consistently $10,000 greater than predicted.



(c) There was a two-year delay in the introduction of the product. After sales began, profits grew as expected.



72. Explain why the graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ about the x-axis.

73. The graph of $y = f(x)$ passes through the points $(0, 1)$, $(1, 2)$, and $(2, 3)$. Find the corresponding points on the graph of $y = f(x + 2) - 1$.

74. ***Think About It*** You can use either of two methods to graph a function: plotting points or translating a parent function as shown in this section. Which method of graphing do you prefer to use for each function? Explain.

(a) $f(x) = 3x^2 - 4x + 1$ (b) $f(x) = 2(x - 1)^2 - 6$

Skills Review

In Exercises 75–82, perform the operation and simplify.

75. $\dfrac{4}{x} + \dfrac{4}{1 - x}$

76. $\dfrac{2}{x + 5} - \dfrac{2}{x - 5}$

77. $\dfrac{3}{x - 1} - \dfrac{2}{x(x - 1)}$

78. $\dfrac{x}{x - 5} + \dfrac{1}{2}$

79. $(x - 4)\left(\dfrac{1}{\sqrt{x^2 - 4}}\right)$

80. $\left(\dfrac{x}{x^2 - 4}\right)\left(\dfrac{x^2 - x - 2}{x^2}\right)$

81. $(x^2 - 9) \div \left(\dfrac{x + 3}{5}\right)$

82. $\left(\dfrac{x}{x^2 - 3x - 28}\right) \div \left(\dfrac{x^2 + 3x}{x^2 + 5x + 4}\right)$

In Exercises 83 and 84, evaluate the function at the specified values of the independent variable and simplify.

83. $f(x) = x^2 - 6x + 11$

(a) $f(-3)$ (b) $f\left(-\frac{1}{2}\right)$ (c) $f(x - 3)$

84. $f(x) = \sqrt{x + 10} - 3$

(a) $f(-10)$ (b) $f(26)$ (c) $f(x - 10)$

In Exercises 85–88, find the domain of the function.

85. $f(x) = \dfrac{2}{11 - x}$

86. $f(x) = \dfrac{\sqrt{x - 3}}{x - 8}$

87. $f(x) = \sqrt{81 - x^2}$

88. $f(x) = \sqrt[3]{4 - x^2}$

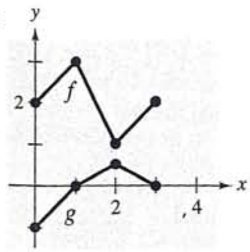
1.8 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. Two functions f and g can be combined by the arithmetic operations of ________, ________, ________, and ________ to create new functions.
2. The ________ of the function f with g is $(f \circ g)(x) = f(g(x))$.
3. The domain of $(f \circ g)$ is all x in the domain of g such that ________ is in the domain of f.
4. To decompose a composite function, look for an ________ function and an ________ function.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, use the graphs of f and g to graph $h(x) = (f + g)(x)$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

1.


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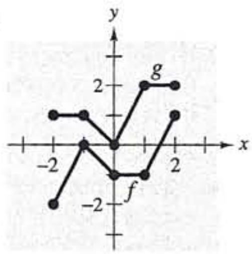

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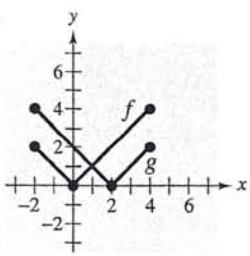

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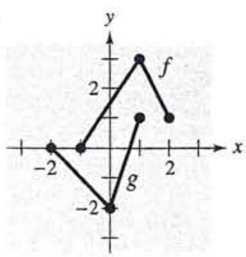


In Exercises 5–12, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

5. $f(x) = x + 2,\quad g(x) = x - 2$
6. $f(x) = 2x - 5,\quad g(x) = 2 - x$
7. $f(x) = x^2,\quad g(x) = 4x - 5$
8. $f(x) = 2x - 5,\quad g(x) = 4$
9. $f(x) = x^2 + 6,\quad g(x) = \sqrt{1 - x}$
10. $f(x) = \sqrt{x^2 - 4},\quad g(x) = \dfrac{x^2}{x^2 + 1}$
11. $f(x) = \dfrac{1}{x},\quad g(x) = \dfrac{1}{x^2}$
12. $f(x) = \dfrac{x}{x + 1},\quad g(x) = x^3$

In Exercises 13–24, evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

13. $(f + g)(2)$
14. $(f - g)(-1)$
15. $(f - g)(0)$
16. $(f + g)(1)$
17. $(f - g)(3t)$
18. $(f + g)(t - 2)$
19. $(fg)(6)$
20. $(fg)(-6)$
21. $\left(\dfrac{f}{g}\right)(5)$
22. $\left(\dfrac{f}{g}\right)(0)$
23. $\left(\dfrac{f}{g}\right)(-1) - g(3)$
24. $(fg)(5) + f(4)$

In Exercises 25–28, graph the functions f, g, and $f + g$ on the same set of coordinate axes.

25. $f(x) = \frac{1}{2}x,\quad g(x) = x - 1$
26. $f(x) = \frac{1}{3}x,\quad g(x) = -x + 4$
27. $f(x) = x^2,\quad g(x) = -2x$
28. $f(x) = 4 - x^2,\quad g(x) = x$

Graphical Reasoning **In Exercises 29 and 30, use a graphing utility to graph f, g, and $f + g$ in the same viewing window. Which function contributes most to the magnitude of the sum when $0 \le x \le 2$? Which function contributes most to the magnitude of the sum when $x > 6$?**

29. $f(x) = 3x,\quad g(x) = -\dfrac{x^3}{10}$
30. $f(x) = \dfrac{x}{2},\quad g(x) = \sqrt{x}$

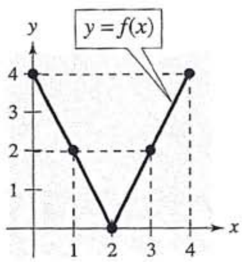
In Exercises 31–34, find (a) $f \circ g$, (b) $g \circ f$, and (c) $f \circ f$.

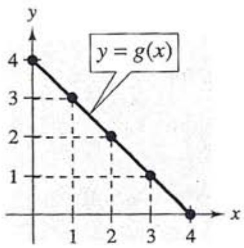
31. $f(x) = x^2,\quad g(x) = x - 1$
32. $f(x) = 3x + 5,\quad g(x) = 5 - x$
33. $f(x) = \sqrt[3]{x - 1},\quad g(x) = x^3 + 1$
34. $f(x) = x^3,\quad g(x) = \dfrac{1}{x}$

In Exercises 35–42, find (a) $f \circ g$ and (b) $g \circ f$. Find the domain of each function and each composite function.

35. $f(x) = \sqrt{x+4}$, $g(x) = x^2$
36. $f(x) = \sqrt[3]{x-5}$, $g(x) = x^3 + 1$
37. $f(x) = x^2 + 1$, $g(x) = \sqrt{x}$
38. $f(x) = x^{2/3}$, $g(x) = x^6$
39. $f(x) = |x|$, $g(x) = x + 6$
40. $f(x) = |x - 4|$, $g(x) = 3 - x$
41. $f(x) = \dfrac{1}{x}$, $g(x) = x + 3$
42. $f(x) = \dfrac{3}{x^2 - 1}$, $g(x) = x + 1$

In Exercises 43–46, use the graphs of f and g to evaluate the functions.





43. (a) $(f+g)(3)$ (b) $(f/g)(2)$
44. (a) $(f-g)(1)$ (b) $(fg)(4)$
45. (a) $(f \circ g)(2)$ (b) $(g \circ f)(2)$
46. (a) $(f \circ g)(1)$ (b) $(g \circ f)(3)$

In Exercises 47–54, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

47. $h(x) = (2x+1)^2$
48. $h(x) = (1-x)^3$
49. $h(x) = \sqrt[3]{x^2 - 4}$
50. $h(x) = \sqrt{9 - x}$
51. $h(x) = \dfrac{1}{x+2}$
52. $h(x) = \dfrac{4}{(5x+2)^2}$
53. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$
54. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

55. ***Stopping Distance*** The research and development department of an automobile manufacturer has determined that when a driver is required to stop quickly to avoid an accident, the distance (in feet) the car travels during the driver's reaction time is given by $R(x) = \frac{3}{4}x$, where x is the speed of the car in miles per hour. The distance (in feet) traveled while the driver is braking is given by $B(x) = \frac{1}{15}x^2$. Find the function that represents the total stopping distance T. Graph the functions R, B, and T on the same set of coordinate axes for $0 \le x \le 60$.

56. ***Sales*** From 2000 to 2005, the sales R_1 (in thousands of dollars) for one of two restaurants owned by the same parent company can be modeled by

$$R_1 = 480 - 8t - 0.8t^2, \qquad t = 0, 1, 2, 3, 4, 5$$

where $t = 0$ represents 2000. During the same six-year period, the sales R_2 (in thousands of dollars) for the second restaurant can be modeled by

$$R_2 = 254 + 0.78t, \qquad t = 0, 1, 2, 3, 4, 5.$$

(a) Write a function R_3 that represents the total sales of the two restaurants owned by the same parent company.

(b) Use a graphing utility to graph R_1, R_2, and R_3 in the same viewing window.

57. ***Vital Statistics*** Let $b(t)$ be the number of births in the United States in year t, and let $d(t)$ represent the number of deaths in the United States in year t, where $t = 0$ corresponds to 2000.

(a) If $p(t)$ is the population of the United States in year t, find the function $c(t)$ that represents the percent change in the population of the United States.

(b) Interpret the value of $c(5)$.

58. ***Pets*** Let $d(t)$ be the number of dogs in the United States in year t, and let $c(t)$ be the number of cats in the United States in year t, where $t = 0$ corresponds to 2000.

(a) Find the function $p(t)$ that represents the total number of dogs and cats in the United States.

(b) Interpret the value of $p(5)$.

(c) Let $n(t)$ represent the population of the United States in year t, where $t = 0$ corresponds to 2000. Find and interpret

$$h(t) = \frac{p(t)}{n(t)}.$$

59. ***Military Personnel*** The total numbers of Army personnel (in thousands) A and Navy personnel (in thousands) N from 1990 to 2002 can be approximated by the models

$$A(t) = 3.36t^2 - 59.8t + 735$$

and

$$N(t) = 1.95t^2 - 42.2t + 603$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: Department of Defense)

(a) Find and interpret $(A + N)(t)$. Evaluate this function for $t = 4$, 8, and 12.

(b) Find and interpret $(A - N)(t)$. Evaluate this function for $t = 4$, 8, and 12.

60. ***Sales*** The sales of exercise equipment E (in millions of dollars) in the United States from 1997 to 2003 can be approximated by the function

$$E(t) = 25.95t^2 - 231.2t + 3356$$

and the U.S. population P (in millions) from 1997 to 2003 can be approximated by the function

$$P(t) = 3.02t + 252.0$$

where t represents the year, with $t = 7$ corresponding to 1997. (Source: National Sporting Goods Association, U.S. Census Bureau)

(a) Find and interpret $h(t) = \dfrac{E(t)}{P(t)}$.

(b) Evaluate the function in part (a) for $t = 7$, 10, and 12.

Model It

61. ***Health Care Costs*** The table shows the total amounts (in billions of dollars) spent on health services and supplies in the United States (including Puerto Rico) for the years 1995 through 2001. The variables y_1, y_2, and y_3 represent out-of-pocket payments, insurance premiums, and other types of payments, respectively. (Source: Centers for Medicare and Medicaid Services)

Year	y_1	y_2	y_3
1995	146.2	329.1	44.8
1996	152.0	344.1	48.1
1997	162.2	359.9	52.1
1998	175.2	382.0	55.6
1999	184.4	412.1	57.8
2000	194.7	449.0	57.4
2001	205.5	496.1	57.8

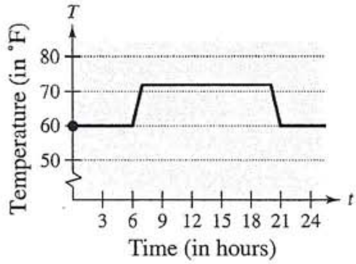
(a) Use the *regression* feature of a graphing utility to find a linear model for y_1 and quadratic models for y_2 and y_3. Let $t = 5$ represent 1995.

(b) Find $y_1 + y_2 + y_3$. What does this sum represent?

(c) Use a graphing utility to graph y_1, y_2, y_3, and $y_1 + y_2 + y_3$ in the same viewing window.

(d) Use the model from part (b) to estimate the total amounts spent on health services and supplies in the years 2008 and 2010.

62. ***Graphical Reasoning*** An electronically controlled thermostat in a home is programmed to lower the temperature automatically during the night. The temperature in the house T (in degrees Fahrenheit) is given in terms of t, the time in hours on a 24-hour clock (see figure).



(a) Explain why T is a function of t.

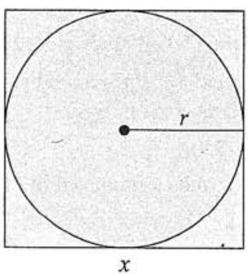
(b) Approximate $T(4)$ and $T(15)$.

(c) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t - 1)$. How does this change the temperature?

(d) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t) - 1$. How does this change the temperature?

(e) Write a piecewise-defined function that represents the graph.

63. ***Geometry*** A square concrete foundation is prepared as a base for a cylindrical tank (see figure).

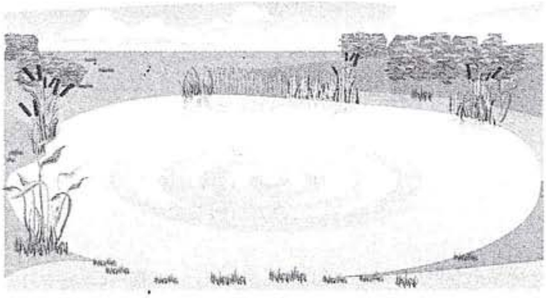


(a) Write the radius r of the tank as a function of the length x of the sides of the square.

(b) Write the area A of the circular base of the tank as a function of the radius r.

(c) Find and interpret $(A \circ r)(x)$.

64. ***Physics*** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles (see figure). The radius r (in feet) of the outer ripple is $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area A of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.



65. ***Bacteria Count*** The number N of bacteria in a refrigerated food is given by

$N(T) = 10T^2 - 20T + 600, \quad 1 \le T \le 20$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$T(t) = 3t + 2, \quad 0 \le t \le 6$

where t is the time in hours.

(a) Find the composition $N(T(t))$ and interpret its meaning in context.

(b) Find the time when the bacterial count reaches 1500.

66. ***Cost*** The weekly cost C of producing x units in a manufacturing process is given by

$C(x) = 60x + 750.$

The number of units x produced in t hours is given by

$x(t) = 50t.$

(a) Find and interpret $(C \circ x)(t)$.

(b) Find the time that must elapse in order for the cost to increase to \$15,000.

67. ***Salary*** You are a sales representative for a clothing manufacturer. You are paid an annual salary, plus a bonus of 3% of your sales over \$500,000. Consider the two functions given by

$f(x) = x - 500{,}000$ and $g(x) = 0.03x.$

If x is greater than \$500,000, which of the following represents your bonus? Explain your reasoning.

(a) $f(g(x))$ (b) $g(f(x))$

68. ***Consumer Awareness*** The suggested retail price of a new hybrid car is p dollars. The dealership advertises a factory rebate of \$2000 and a 10% discount.

(a) Write a function R in terms of p giving the cost of the hybrid car after receiving the rebate from the factory.

(b) Write a function S in terms of p giving the cost of the hybrid car after receiving the dealership discount.

(c) Form the composite functions $(R \circ S)(p)$ and $(S \circ R)(p)$ and interpret each.

(d) Find $(R \circ S)(20{,}500)$ and $(S \circ R)(20{,}500)$. Which yields the lower cost for the hybrid car? Explain.

Synthesis

True or False? **In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.**

69. If $f(x) = x + 1$ and $g(x) = 6x$, then

$(f \circ g)(x) = (g \circ f)(x).$

70. If you are given two functions $f(x)$ and $g(x)$, you can calculate $(f \circ g)(x)$ if and only if the range of g is a subset of the domain of f.

71. ***Proof*** Prove that the product of two odd functions is an even function, and that the product of two even functions is an even function.

72. ***Conjecture*** Use examples to hypothesize whether the product of an odd function and an even function is even or odd. Then prove your hypothesis.

Skills Review

Average Rate of Change **In Exercises 73–76, find the difference quotient**

$$\frac{f(x+h) - f(x)}{h}$$

and simplify your answer.

73. $f(x) = 3x - 4$

74. $f(x) = 1 - x^2$

75. $f(x) = \dfrac{4}{x}$

76. $f(x) = \sqrt{2x + 1}$

In Exercises 77–80, find an equation of the line that passes through the given point and has the indicated slope. Sketch the line.

77. $(2, -4), m = 3$

78. $(-6, 3), m = -1$

79. $(8, -1), m = -\frac{3}{2}$

80. $(7, 0), m = \frac{5}{7}$

1.9 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. If the composite functions $f(g(x)) = x$ and $g(f(x)) = x$ then the function g is the ________ function of f.
2. The domain of f is the ________ of f^{-1}, and the ________ of f^{-1} is the range of f.
3. The graphs of f and f^{-1} are reflections of each other in the line ________.
4. A function f is ________ if each value of the dependent variable corresponds to exactly one value of the independent variable.
5. A graphical test for the existence of an inverse function of f is called the _______ Line Test.

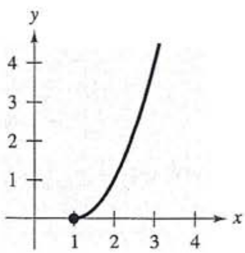
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

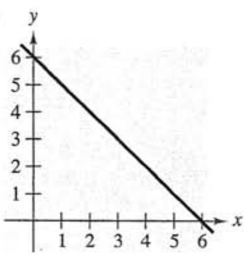
1. $f(x) = 6x$
2. $f(x) = \frac{1}{3}x$
3. $f(x) = x + 9$
4. $f(x) = x - 4$
5. $f(x) = 3x + 1$
6. $f(x) = \dfrac{x-1}{5}$
7. $f(x) = \sqrt[3]{x}$
8. $f(x) = x^5$

In Exercises 9–12, match the graph of the function with the graph of its inverse function. [The graphs of the inverse functions are labeled (a), (b), (c), and (d).]

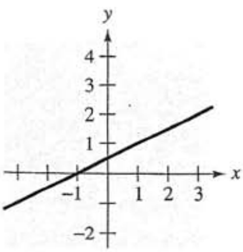
(a)



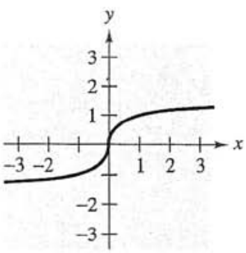
(b)



(c)



(d)



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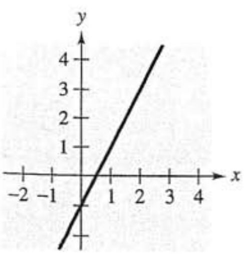

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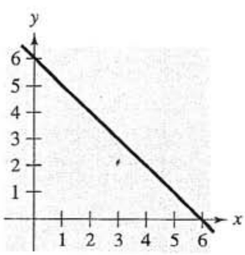

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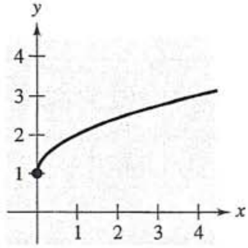

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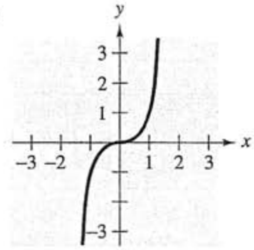


In Exercises 13–24, show that f and g are inverse functions (a) algebraically and (b) graphically.

13. $f(x) = 2x,\quad g(x) = \dfrac{x}{2}$
14. $f(x) = x - 5,\quad g(x) = x + 5$
15. $f(x) = 7x + 1,\quad g(x) = \dfrac{x-1}{7}$
16. $f(x) = 3 - 4x,\quad g(x) = \dfrac{3-x}{4}$
17. $f(x) = \dfrac{x^3}{8},\quad g(x) = \sqrt[3]{8x}$
18. $f(x) = \dfrac{1}{x},\quad g(x) = \dfrac{1}{x}$
19. $f(x) = \sqrt{x-4},\quad g(x) = x^2 + 4,\ x \ge 0$
20. $f(x) = 1 - x^3,\quad g(x) = \sqrt[3]{1-x}$
21. $f(x) = 9 - x^2,\ x \ge 0,\quad g(x) = \sqrt{9-x},\ x \le 9$
22. $f(x) = \dfrac{1}{1+x},\ x \ge 0,\quad g(x) = \dfrac{1-x}{x},\ 0 < x \le 1$
23. $f(x) = \dfrac{x-1}{x+5},\quad g(x) = -\dfrac{5x+1}{x-1}$
24. $f(x) = \dfrac{x+3}{x-2},\quad g(x) = \dfrac{2x+3}{x-1}$

In Exercises 25 and 26, does the function have an inverse function?

25.

x	-1	0	1	2	3	4
$f(x)$	-2	1	2	1	-2	-6

26.

x	-3	-2	-1	0	2	3
$f(x)$	10	6	4	1	-3	-10

In Exercises 27 and 28, use the table of values for $y = f(x)$ to complete a table for $y = f^{-1}(x)$.

27.

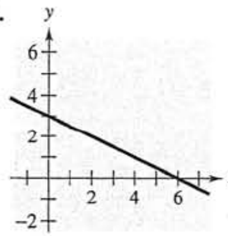
x	-2	-1	0	1	2	3
$f(x)$	-2	0	2	4	6	8

28.

x	-3	-2	-1	0	1	2
$f(x)$	-10	-7	-4	-1	2	5

In Exercises 29–32, does the function have an inverse function?

29.



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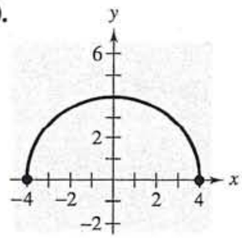

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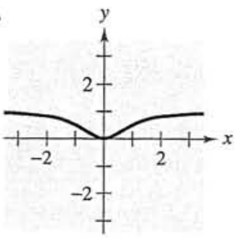

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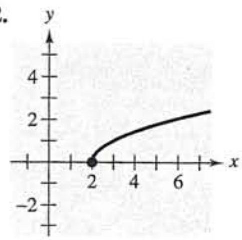


In Exercises 33–38, use a graphing utility to graph the function, and use the Horizontal Line Test to determine whether the function is one-to-one and so has an inverse function.

33. $g(x) = \dfrac{4 - x}{6}$

34. $f(x) = 10$

35. $h(x) = |x + 4| - |x - 4|$

36. $g(x) = (x + 5)^3$

37. $f(x) = -2x\sqrt{16 - x^2}$

38. $f(x) = \frac{1}{8}(x + 2)^2 - 1$

In Exercises 39–54, (a) find the inverse function of f, (b) graph both f and f^{-1} on the same set of coordinate axes, (c) describe the relationship between the graphs of f and f^{-1}, and (d) state the domain and range of f and f^{-1}.

39. $f(x) = 2x - 3$

40. $f(x) = 3x + 1$

41. $f(x) = x^5 - 2$

42. $f(x) = x^3 + 1$

43. $f(x) = \sqrt{x}$

44. $f(x) = x^2, \quad x \ge 0$

45. $f(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$

46. $f(x) = x^2 - 2, \quad x \le 0$

47. $f(x) = \dfrac{4}{x}$

48. $f(x) = -\dfrac{2}{x}$

49. $f(x) = \dfrac{x + 1}{x - 2}$

50. $f(x) = \dfrac{x - 3}{x + 2}$

51. $f(x) = \sqrt[3]{x - 1}$

52. $f(x) = x^{3/5}$

53. $f(x) = \dfrac{6x + 4}{4x + 5}$

54. $f(x) = \dfrac{8x - 4}{2x + 6}$

In Exercises 55–68, determine whether the function has an inverse function. If it does, find the inverse function.

55. $f(x) = x^4$

56. $f(x) = \dfrac{1}{x^2}$

57. $g(x) = \dfrac{x}{8}$

58. $f(x) = 3x + 5$

59. $p(x) = -4$

60. $f(x) = \dfrac{3x + 4}{5}$

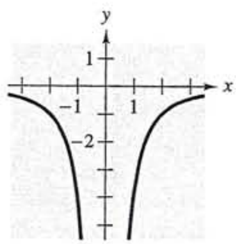
61. $f(x) = (x + 3)^2, \quad x \ge -3$

62. $q(x) = (x - 5)^2$

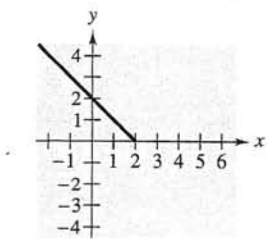
63. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$

64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$

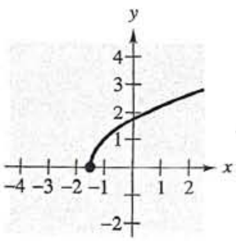
65. $h(x) = -\dfrac{4}{x^2}$



66. $f(x) = |x - 2|, \quad x \le 2$



67. $f(x) = \sqrt{2x + 3}$



68. $f(x) = \sqrt{x - 2}$